

BANK ASSET AND LIABILITY MANAGEMENT
-- A FUZZY MATHEMATICAL PROGRAMMING MODEL

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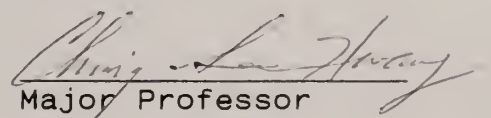
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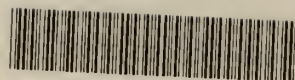
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CHAPTER 1

INTRODUCTION

Management science models have been formulated to help bank executives, decision makers, in handling problems and making their financial decisions that arise in a variety of banking areas. The problems that the decision makers usually face in a commercial bank are to develop the structure of each balance sheet position and to make allowance for complex interactions of sources and uses, particularly in two dimensions of money and time [30].

In managing bank balance-sheet position, profit, liquidity and its associated risk are the main concerns of financial strategy. The more the profit a bank earns, the higher the risk it faces and the less the liquidity it has. The optimal trade-off relationship between risk, return and liquidity becomes a complex problem for decision makers.

In a state of certainty, banker would provide himself with cash or maturing securities sufficient to satisfy all transactions requirements and then invest the remainder of his portfolio in the highest yielding asset. Unfortunately, the environment in which he operates is one characterized by risk and uncertainty. His decisions are constrained by a lack of knowledge about their outcomes.

The determination of the optimal size and composition of a bank's asset, liability and capital accounts over a multiperiod planning horizon is subject to the inherent uncertainty of banking, the cash flows, cost of funds and returns on investments and economic conditions. It also relies much on decision maker's subjective judgment.

Many approaches that incorporate mathematical programming proposed by researchers attempt to deal with the problem of uncertainty, yet none can handle the problem well since the real-world situations are not often crisp and deterministic thus cannot be described precisely.

Instead of expressing the imprecision with randomness as in most stochastic models, fuzzy set theory [40], a theory of graded concepts, regards everything as a matter of degree and does not have a crisp boundary which separate those objects which belong to a class from those which do not. This is especially useful for handling the uncertainty of areas in which human judgment, evaluation, and decision are important [1] [42].

In this study, we propose a model which (1) integrates aspects of the existing bank management models; (2) incorporates the fuzzy set theory to handle the inherent uncertainty of banking problems and decision maker's subjective judgment and (3) determines optimal trade-off through a interactive decision process.

The subsequent chapters are organized as follows:

Chapter 2 focuses on the concepts of bank asset and liability management and discusses several alternative approaches for managing bank asset and liability. Chapter 3 provides surveys of the models for managing bank balance sheet and uses Interactive Sequential Goal Programming II (ISGP II) [12] to solve a deterministic model to obtain an optimal trade-off for the size and composition of bank assets and liabilities and their associated risk and liquidity. This trade-off analysis has not been examined systematically in previous studies. In chapter 4, we further utilize the concepts of fuzzy set theory to reformulate the deterministic model examined in chapter 3 in order to deal with the uncertainty inherent in banking management. This model can be used to help decision makers to make their optimal planning in the face of fuzzy environment. In the last chapter, chapter 5, important findings and conclusions in this study are summarized and directions for further research presented.

CHAPTER 2

BANK ASSET AND LIABILITY MANAGEMENT

Commercial banks are intermediaries that provide financial services to individuals and institutions. Bank asset and liability management, a fund management approach, refers to the creation and implementation of strategies to meet a bank's profit and risk objectives. The purpose of this chapter is to describe what bank asset and liability management is and to show the factors and approaches which affect the decisions made by the bank's senior executives.

2.1 Historical Development

There have been several steps in the development of asset and liability management [16]. During the 1950s, with an abundance of relatively stable, low cost funds, extremely conservative investment planning, a concept of "asset allocation" was emphasized and spreads were achieved without the competition and severe pressure. During the 1960s, a highly economic growth took place as a result of technological innovations which led to the increasing need for funds. During 1970s, emphasis shifted to profitability management, from the point of view of income statement management. As the interest rates volatiled heavily during early 1980s and the cost of funds and inherent risk increased significantly, the profitability can only be achieved by analyzing the information on both side of balance sheet and implementing an approach for managing the asset and liability for the financial institutions.

2.2 Evaluating Bank Performance

A business buys raw materials and produces goods and services by combining with capital and labor, and then sells these goods and services to others at prices high enough to yield returns above the cost of providing the goods and services -- the cost of raw materials, capital and labor. For a financial institution, the raw material purchased is funds instead of iron, wood, cloth, and the product sold is funds packaged in a usable form instead of steel, furniture and clothing. According to the financial theory, bank management's basic objective should be to maximize the value of the owners' investment in the bank [19]. Generally, higher returns are available if higher risks are taken. The bank's management tries to balance the tradeoffs between the risks and returns by maximizing the owners' investment in the bank.

2.2.1 Measuring the Returns

A commonly used measure of bank performance is net interest margin or interest spread. Net interest margin is calculated as the difference between interest income generated from its asset portfolio and interest expense, the cost of acquired funds, and overhead expenses hold as constant, expressed as a percentage of total assets as shown below:

$$\text{Net Interest Margin} = \frac{\text{Interest Income} - \text{Interest Expenses}}{\text{Total Assets}}$$

Clearly, a bank can increase its net interest margin, as net income if other expenses hold as constant, and hence its profits, by either increasing the yield on its assets or reducing the cost of its acquired funds or both. Higher returns can be earned by investing in assets with greater risk. Fund costs can be reduced by using more demand deposits and equity. Another important measurement of banking returns is the return on equity (net income divided by equity capital).

2.2.2 Measuring the Risks

The risk measurements are related to the return measurements, because a bank must take risks in order to earn adequate returns. There are several kinds of risks that banks will confront and need to minimize. They are liquidity risk or funding risk, capital risk, default risk or credit risk, interest rate risk [19] [34], country risk and foreign exchange risk [10]. Some risks will be faced by both domestic and international banks, last two risks will occur only in international banks. Each of these risks will be described briefly here and some of them will be formulated and shown in the model in subsequent chapters.

Liquidity Risk / Funding Risk:

Liquidity risk refers to the ability of a bank to meet deposit withdrawals and loan requests, cash outflows, from either selling an asset it holds or acquiring an additional liability, cash inflows. Liquidity risk is the risk when the bank's outflows exceed its inflows at a given moment and force the bank to obtain additional cash quickly to cover the negative net cashflow. Generally, the risk arises when the maturity structure of a bank's liability is shorter than the maturity structure of its assets. It also happens when shifting funds from short-term investments to longer-term securities and loans raises the bank's income while exposing the bank to more risk since financial claims with longer maturities are less liquid as they are more difficult and costly to convert to cash. To minimize the liquidity risk, bank should maintain a minimum level of liquid assets and pay close attention to the bank's maturity mismatch position and keep it in acceptable limits.

Capital Risks:

The capital risk of a bank indicates how much asset values may decline before the position of depositors and other creditors suffer losses. Equity capital protects creditors by serving as a buffer to absorb losses, so, the greater the amount of capital, the lower the level of capital risk.

Default Risk / Credit Risk:

Credit risk refers to the possibility that promised interest and principal payments, or both, on securities and loans will not be paid as promised by the borrowers on time and in full. The credit risk is higher if the bank has more medium-quality loans, but returns are usually higher. Bank control credit risk by carefully analyzing the credit worthiness and limiting the credit amount.

Interest Rate Risk:

Interest rate risk refers to fluctuations in net interest margin that occur because asset and liability returns and values change at different rates when interest rates change. This risk arises when the maturity structure of the assets mismatch the maturity structure of the liabilities. When bankers expect interest rates to go up, they may borrow long and lend short (a positive mismatch); if they expect interest rates to go down, they may borrow short and lend long (a negative mismatch). A beginning measurement of this risk is the ratio of interest-sensitive assets to interest-sensitive liabilities. The first decision that bank management must face is either to hedge the rate sensitivity of the balance sheet or to speculate on interest rate movements. Several approaches were developed to control the interest rate risk: forecasting the interest rate movement, simulation, maturity gap management and

duration gap management, these will be discussed latter in this chapter.

Country Risk:

Country risk refers to the risk incurred by international banks lending funds to clients in another country. To minimize their exposure to country risk, bankers determine the maximum amount of loans and limit the mismatch of assets and currency and liabilities.

Foreign Exchange Risk:

Foreign exchange risk is incurred by international banks with foreign currency assets and liabilities. If assets held in that currency are greater (less) than liabilities owed in the currency, the bank will profit if the exchange value of the foreign currency rises (falls) but will suffer a loss if the exchange value falls (rises).

2.2.3 Objectives of Asset and Liability Management

Obviously, bank managemnet would prefer the highest returns for a given level of risks and lowest risks for a given level of returns. A bank must determine its optimal trade-off between risk, return, and liquidity.

2.3 Regulation and Operational Constraints

Bank management is subject to regulations and operational constraints [2] [14] [15] [24] [35] which stipulate bank's activities and protect creditors equity.

2.3.1 Capital Adequacy Ratio

The financial intermediation system in most countries is subject to government intervention. The capital adequacy of banks imply that the primary function of the bank capital is to preserve the bank as viable entity through periods of financial distress. "The ultimate strength of a bank lies in its net worth or capital funds". Such is the consensus of writers on banking, and of bank supervisors. When depositors demand cash, however, the amount of bank capital is analogous to that of required reserves. For the operating bank, capital adequacy is the amount of the bank's liquid assets that determines whether or not a bank can adequately meet deposit withdrawals and the size of the capital accounts. Capital adequacy should be related to the risk factors in a bank's assets rather than to its total deposits. As a result, the standard of capital adequacy gradually shifted over to what has become known as the risk-asset ratio.

There are several definitions of risk assets that may be used in deriving the risk-asset ratio. Risk assets may be defined as total asset less cash, US government

securities, and loans backed or guaranteed by the US government.

2.3.2 Required Reserves

Commercial banks that are members of the Federal Reserve System are required to hold asset reserves equal to a certain percentage of their deposit liabilities. The requirements is limited by law to a certain range for each size of bank and type of deposit. The required reserves are calculated on average deposits over the reserve period.

For making up deficiencies in reserves, bank can sell assets from its portfolio, borrow on the federal funds market, or borrow from the Federal Reserve Bank. The approach chosen depends upon the cost related to interest rate and the length of time the bank thinks it will need for funds.

2.3.3 Pledged Assets

Federal law requires that any bank holding United States Government deposits must provide collateral backing for these deposits in the form of various specified assets.

2.3.4 Real Estate Loans

The constraint on aggregate real estate loans is calculated in terms of the bank's capital or its savings deposits, whichever allows the larger holdings.

2.3.5 Balancing Constraints

These are elements of the portfolio over which the bank has little or no control but which must be included nonetheless. They are (1) holdings of vault cash, (2) holdings of deposits with other banks, (3) cash in process of collection, (4) other assets.

2.4 Approaches for Measuring and Managing Interest Rate Risk

The interest rate risk that financial intermediaries and commercial banks usually run is mainly due to the fact that these institutions hold assets and liabilities with more or less differing interest rate rigidity. There is no interest risk present if a fixed-rate asset is financed by a fixed-rate liability with equal amount and interest term. Interest rate risk arises if either amount or interest term of the asset differs from the amount or interest term of the liability that was used for funding the asset. Interest rate risk also exists if floating-rate assets are financed by fixed-rate liabilities, or if fixed-rate assets are financed by floating-rate liabilities. In this section, four approaches for managing and measuring the interest rate risk are presented [14] [32] [38].

2.4.1 Forecasting

Before incorporate other approaches to managing interest rate risk, some sophisticated statistical techniques have been used to provide forecasts of deposits, loans and interest rates. The statistical technique chosen in bank forecasting models depends on the length of the forecast horizon. In the short run, the univariate time series models may provide excellent forecasts if we look the bank's loans and deposits as determined by environmental

forces and these forces act in a periodic fashion. In the long run, large-scale econometric models are selected and play a useful role in bank financial planning as we look loans and deposits as determined by both supply and demand forces [14].

2.4.2 Simulation

Computer simulation models must be used in conjunction with the forecasting techniques and produce the results in a dynamic or forward-looking way [38]. Just as in any other asset / liability model, simulations require data on the cash flows of the currently booked assets and liabilities. The more accurate the representation of these cash flows, the more informative the simulations. Simulations increase the value of strategic planning and profitability exercises. By varying the forecasts put into the model, a risk-return tradeoff can be derived; by constructing a probability distribution for the occurrence of the events, the potential variance of earnings in addition to the expected earnings for each possible forecast can be estimated. Through this simulation, the bank can develop several different portfolios, along with the estimated return and variance of each. From the enormous output generated by simulation approach, the senior executives of banks can make their decisions in choosing the optimal set of portfolio of bank assets and liabilities.

2.4.3 Maturity Gap or Interest Surplus Analysis

The gap model derives its name from the dollar gap (Gap\$) that is the difference between the dollar amounts of rate-sensitive assets (RSA\$) and rate-sensitive liabilities (RSL\$) [38].

$$\text{Gap\$} = \text{RSA\$} - \text{RSL\$}$$

Four types of information are required in this model: first, the length of time over which net interest income is to be managed, this is so called "gapping period" usually is one year; second, deciding whether to preserve the currently expected net interest income or to attempt to better it; third, specifying an interest rate forecast or a set of probability-weighted forecasts; finally, determining the dollar amounts of the rate-sensitive assets and rate-sensitive liabilities.

Rate-sensitive assets are those that can experience contractual changes in interest rates during the gapping period. All financial assets and liabilities that mature within the gapping period are rate-sensitive. Variable rate assets and liabilities which repriced during the gapping period are also rate-sensitive regardless of their maturity dates. Shown in Figure 2-1 is "gap" as described above.

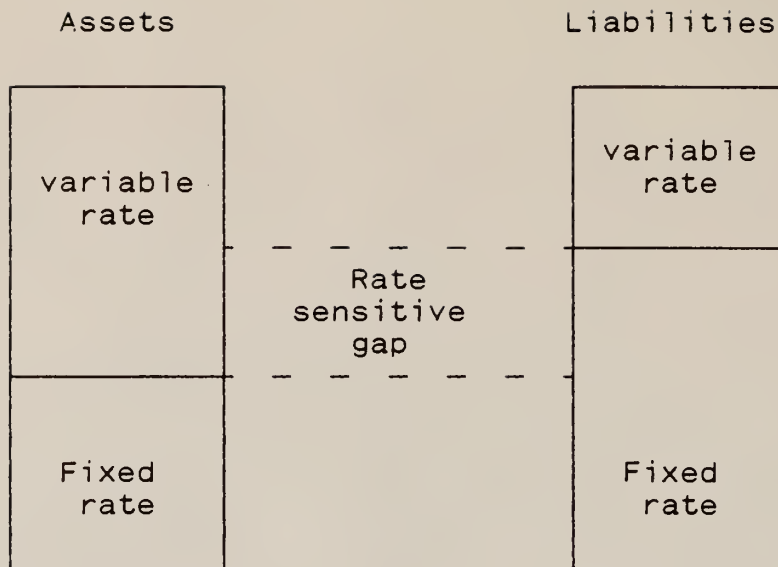


Figure 2-1 Gap of RSA and RSL

Gap analysis defines various pools of funds by rates and balances. It attempts to manage these pools to achieve a satisfactory level of profitability and an acceptable level of risk [32]. The disadvantages of using gap analysis are that the collection of data on all balances by maturity and by rate is no easy matter, and this is a continuing problem when performing gap analysis. In addition, the predict of what interest rates will do in the future is a difficulty in gap analysis as in many other types of asset and liability management analysis.

2.4.4 Duration Gap or Market Value Analysis

The duration gap analysis or market value analysis [31] [6] is a present value technique which measures the

"Duration" or average maturity of assets and liabilities. The duration can be defined by Kaufman as follows [23]: Duration is a measure of the average life of a security. It is computed by multiplying the length of time to each scheduled payment by the ratio of the present value of that payment to the total present value or price of the security and summing, or

$$D = \frac{\sum_{t=1}^n t * PVF_t}{\sum_{t=1}^n PVF_t}$$

where:

D = duration

t = length of time (number of months, years) to the date of payment

PVF_t = present value of the payment (F) made at (t), or $F_t / (1 + i)^t$

$\sum_{t=1}^n$ = summation from the first to the last payment

Duration is a single number that is measured in units of time, months or years. For securities that make only one

payment at maturity, duration is equal to maturity; for all other securities, it is shorter than term to maturity.

This technique, duration gap analysis, avoids the definitional problems of identifying pools of funds and uses the sensitivity of asset and liability values of interest-rate changes as input to develop management strategies. These strategies exploit the combined sensitivity (duration gap) such that a bank is immunized against interest-rate if the duration gap equals zero. This means to match the duration maturities of cash outflows with cash inflows. If the direction of interest rates can be forecasted, a nonzero duration gap strategy is appropriate. For example, if the duration of assets are later than the duration of liabilities, that means the assets have to be refinanced on average before they mature, in this case, the bank would then be at risk in a rising interest rate environment.

Duration analysis can be performed on a number of target accounts [23] [38], such as market value of equity, Economic leverage, total return on equity, net interest income etc. This permits risk measurement for balance sheet items and income statement items. It also reveals the largest potential number of asset and liability choices capable of moving current risk exposures to more desired levels.

The accuracy of risk assessments using duration analysis relies on the fulfillment of certain assumptions,

the assumptions of flat yield curve. Matching asset and liability risks using durations but not cash flows requires making frequent adjustment of asset and liability positions to minimize the problem of the asset duration drifting away, as time passes, from the liability duration.

Duration gap models provide more flexibility in risk control than other approaches do. It will become the leading technique because it accurately assesses the risk associated with the management of the net interest margin [32].

CHAPTER 3

DETERMINISTIC AND STOCHASTIC MODEL

A number of mathematical programming techniques have been developed to help evaluate the economic trade-offs between important decision variables as they have distinct advantages over simulation models. In this chapter, an overview of these mathematical programming techniques are presented, then an approach, Interactive Sequential Goal Programming II (ISGP II) [12], which incorporates multiple objective decision making is used to determine the optimal trade-offs between competing and conflicting objectives such as maximization of returns and minimization of risks associated with alternative portfolio combinations of a commercial bank. An example defining as deterministic model, interest rates of portfolios are known and deposit flows are fixed, is solved to provide information which helps the decision makers making their decisions and achieving their maximum satisfactions.

3.1 Constructions of Models

Booth and Bessler [6] pointed out that the asset and liability management models encompass a wide variety of construction schemes, such as (1) single vs multiple goal, (2) linear vs nonlinear, (3) deterministic vs stochastic, (4) single vs multiple period and (5) application (many variables and constraints) vs prototype (few variables and constraints). Kusy and Ziemba [25] suggested that an ideal operational model should contain the following features: (1) multiperiodicity that incorporates: changing yield spreads across time, transaction costs associated with selling assets prior to maturity, and the synchronization of cash flows across time by matching maturity of assets with expected cash outflows; (2) simultaneous considerations of assets and liabilities to satisfy basic accounting principles and match the liquidity of assets and liabilities; (3) transaction costs that incorporate brokerage fees and other expenses incurred in buying and selling securities; (4) uncertainty of cash flows that incorporates the uncertainty inherent in the depositors' withdrawal claims and deposits (to ensure that the structure of the asset portfolio gives the bank the capacity to meet these claims); (5) the incorporation of uncertain interest rates into the decision-making process to avoid lending and borrowing decisions that may ultimately be detrimental to the financial well-being of the bank; and

(6) legal and policy constraints appropriate to the bank's operating environment. For the simplicity of the problem, some features of this ideal model will not be included when formulate a new model.

3.2 Review of Mathematical Programming Techniques

Many different management science techniques, such as forecasting, simulation and mathematical programming, have been applied to formulate bank asset and liability management models [14]. A properly formulated mathematical programming models can determine the optimal size and composition of a bank's asset, liability and capital accounts over a multiperiod planning horizon, while simulation models only allow management to evaluate a small set of alternatives.

3.2.1 Deterministic models --- Single Objective

In a pioneering article of Chambers and Charnes [11] shows how to solve a simple balance sheet management problem with an intertemporal linear programming (LP) model. They assumed that all the economic situations are under certainty and determined the optimal size and composition of assets and liabilities by maximizing profits subject to the constraints in order to meet the minimum reserve requirements and the capital adequacy tests of the Federal Reserve Board [2] [37] (see Appendix A). All of the deterministic models and many of the stochastic models follow the way they did to construct a new model. Cohen and Hammer [13] extended to include the bankers' policy constraints, market constraints and feedback between a bank's loans and deposits. These models using linear

programming do not consider the uncertainty of economic situations.

The general form of the LP model with m constraints and n variables ($n \geq m$) is:

$$\text{Max / Min } Z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

s.t.

$$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n = b_1$$

$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n = b_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot$$

$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n = b_m$$

$$\underline{x}, \underline{b} \geq \underline{0}$$

3.2.2 Deterministic models --- Multiple Objectives

There are two approaches deal with multiple objectives. One is goal programming (GP), which recognizes the existence of multiple and possible conflicting goals.

Fortson and Dince [18] formulate a model concerned with four specific goals: profit, the ratio of loans to deposits, capital adequacy and liquidity; Tayi and Leonard [37] use profit, risk asset ratio and capital adequacy ratio as goals; Booth and Bessler [6] set profit and duration gap as goals.

The objective of GP model is simultaneously to minimize

the weighted sum of the absolute value of the deviations from pre-stated goal values for the objectives. The GP formulation is:

$$\text{Min } \left[\sum_{j=1}^k (d_j^- + d_j^+) \right]^{1/p}$$

s.t.

$$g_i(\underline{x}) \leq 0, \quad i = 1, \dots, m$$

$$f_j(\underline{x}) + d_j^- - d_j^+ = b_j, \quad j = 1, \dots, k$$

$$d_j^- * d_j^+ = 0, \quad j = 1, \dots, k$$

$$\underline{x}, \underline{d}^-, \underline{d}^+ \geq 0$$

where d_j^+ and d_j^- are positive and negative deviations, respectively, of objective j from its goal.

GP is one of the earliest methods in dealing with multiple objectives, it is an extension of linear programming and is advantageous for its simple formulation, simple algorithm, and easy usage. It requires the decision maker to indicate the initial goals for each objective before solving the problems. Lin [27] concluded four disadvantages of using GP: (1) the difficulty of setting the initial goals; (2) the possibility of getting the dominated solutions; (3) Incompatibility with utility preferences; (4) "absolutely more important" is not common.

Another approach for solving multiple objectives problems is multiple objective linear programming (MOLP). Sealey [33], Eatman and Sealey [17] show a model with three objectives: maximization of net after-tax profit, minimization of the ratio of risk assets to capital and minimization of the ratio of the capital adequacy. They use the algorithm developed by Zeleny [41] to obtain all sets of nondominated extreme point solutions [37]. A trade-off is made by decision maker while considering competing and conflicting objectives. A more detail of MOLP problems will be discussed in section 3.3.

3.2.3 Stochastic Models

The stochastic models, including the use of chance-constrained programming, dynamic programming, sequential decision theory and stochastic linear programming with simple recourse, have achieved little success in bank balance sheet management due to inherent computational difficulties, oversimplifications and unfamiliarity of the models [25].

The chance-constrained models express future deposits and loan repayments as joint, normally distributed random variables, however, multiperiod models raise conceptual difficulties in this approach.

The dynamic programming models are dynamic and account for the uncertainty of the problem but are of limited use in

practice.

The sequential decision analysis proposed by Wolf [39] find an optimal solution through the use of implicit enumeration [25]. The stochastic linear programming with simple recourse explicitly characterizes each realization of the random variables by a constraint and leads to large problems in realistic situations. Cohen and Thore [15], Booth [4] [5], Bradley and Crane [7] and Kusy and Ziemba [25] introduced two-stage models which consists of two discrete time periods, first period is short and forecastable while second period is long and uncertain, and some economic situations such as definitional, accounting, legal and financial constraints. Korhonen [24] combines two-stage model with goal programming as two-stage goal programming model.

Another approach is proposed by Brodt [8] [9] [10] called MIN-MAD model which is a multiperiod linear programming under uncertainty model based on Markowitz portfolio theory [29] and on the models developed by Cohen and Thore [15] and Bradley and Crane [7].

3.3 Multiple Objective Decision Making (MODM)

Except for special situations in which optimization of a single objective is sufficient, decision making generally involves multiple and ordinarily conflicting objectives.

Multiple objectives problems in decision making process deal with multiple conflicting and competing objectives. Instead of using DM's intuitive judgment, some mathematical approaches are applied to help DM making rational decision. Since there is no single optimal solution for MODM problems, a set of "nondominated" solutions is sought through a trade-off process.

3.3.1 Formulation of MODM Problems

The general formulation of MODM problems are defined as:

$$\text{Max / Min } \{ f_1(\underline{x}), f_2(\underline{x}), \dots, f_k(\underline{x}) \}$$

s.t.

$$\underline{x} \in X$$

where

$$X = \{ \underline{x} \mid g_i(\underline{x}) \{ \geq, =, \leq, \} 0; i = 1, 2, \dots, m \}$$

$f_j(\underline{x})$: the benefit objective for maximization, $j \in J$

$f_i(\underline{x})$: the cost objective for minimization, $i \in I$

3.3.2 Classification of MCDM Problems and Solution Methods

Hwang and Masud [20] collected 19 solution methods and classified them into four categories according to the types of information needed. Figure 3-1 shows a taxonomy of these methods.

3.3.3 MODM Solutions

There are different kinds of solution should be taken into account while dealing with multiple objective problems such as optimal, positive ideal and negative ideal, nondominated, satisficing and preferred solution. An objective function space representation of two objectives and these MODM solutions are depicted in Figure 3-2.

Optimal solution:

Optimal solution is one which results in the maximum value of each objective simultaneously. That is, $\underline{x}^*$ is an optimal solution to the problem if $\underline{x}^* \in X$ and $\underline{f}(\underline{x}^*) \geq \underline{f}(\underline{x})$ for all $\underline{x} \in X$. Since the objectives are conflicting to each other, no optimal solution exists in MODM problems.

Nondominated Solution:

A feasible solution in MODM is nondominated if there exists no other feasible solution that will yield an improvement in one objective without causing a degradation in at least one other objective.

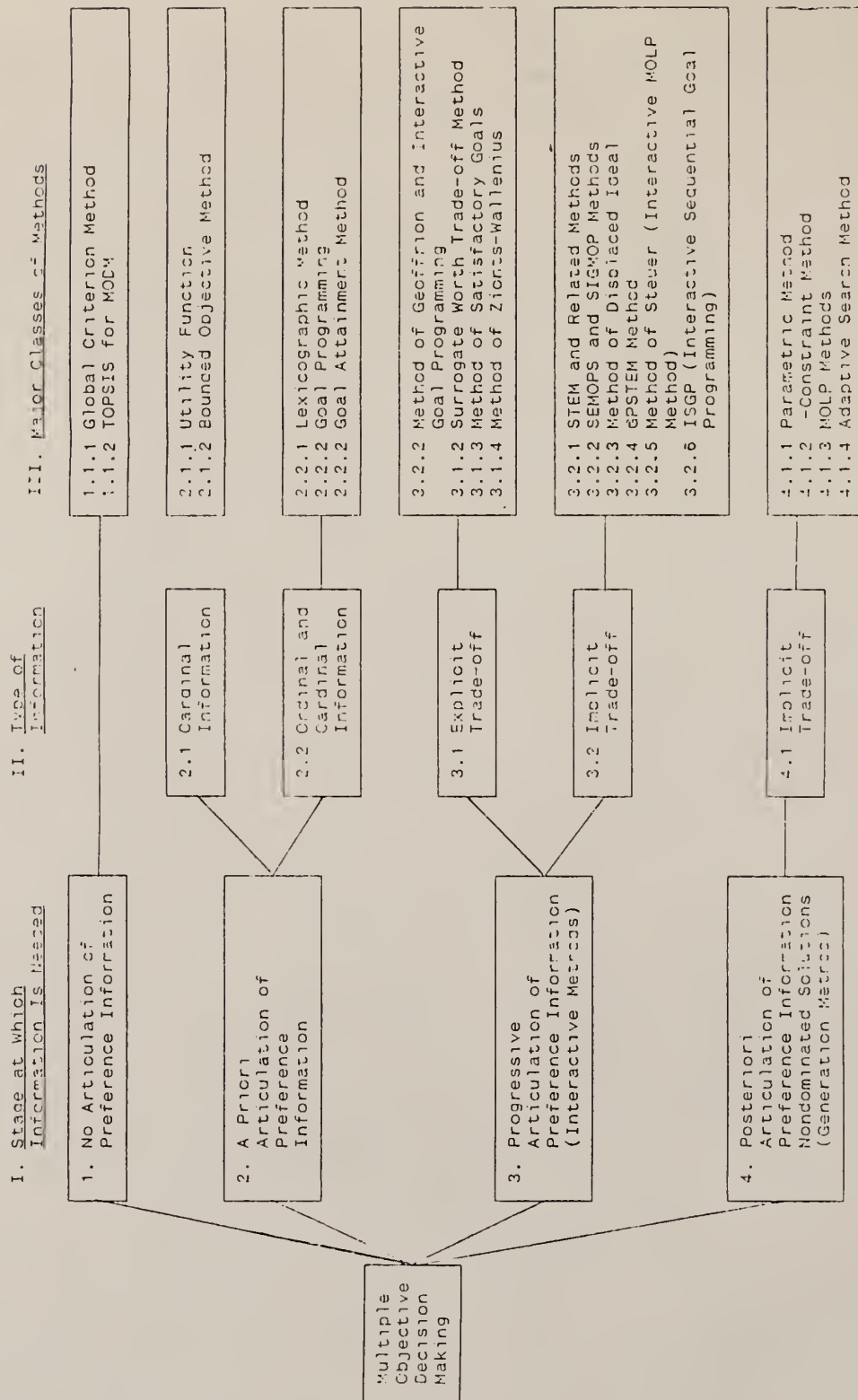


Figure 3-1 A Taxonomy of Methods for Multiple Objective Decision Making (Hwang & Masud)

MCDM SOLUTIONS

- * OPTIMAL SOLUTION
- * IDEAL SOLUTION -- utopia point, superior solution
- * NONDOMINATED SOLUTION -- noninferior solution, efficient solution,
a set of admissible alternatives.
Pareto-optimal solution
- * SATISFICING SOLUTION
- * PREFERRED SOLUTION

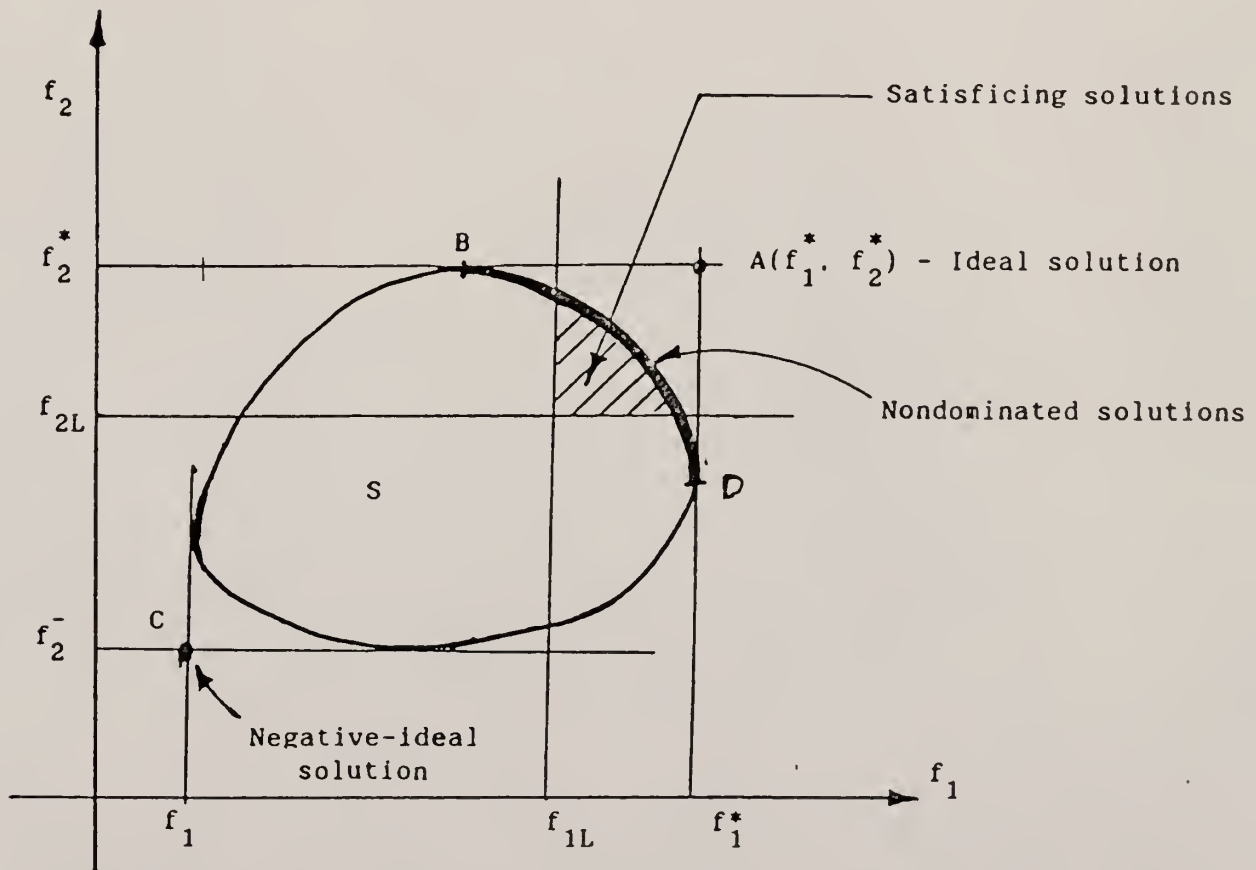


Figure 3-2 Objective function space representation of 2 objectives.

Positive-Ideal Solution:

The positive-ideal solution is the one that optimizes each objective function simultaneously:

$$\text{Max } f_j(\underline{x}), j \in J$$

$$\text{Min } f_j(\underline{x}), i \in I$$

s.t.

$$\underline{x} \in X$$

An ideal solution can be defined as $A^* = (f_1^*, f_2^*, \dots, f_k^*)$, where f_1^* is a feasible and optimal value for the 1th objective function. The PIS payoff table is given as:

	f_1	f_2	.	.	.	f_k	
f_1	f_1^*	f_{12}	.	.	.	f_{1k}	(diagonal elements are the PIS)
f_2	f_{21}	f_2^*	.	.	.	f_{2k}	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
f_k	f_{k1}	f_{k2}	.	.	.	f_k^*	

Negative-Ideal Solution:

Similarly, the concept of a negative-ideal solution, or the worst possible solution for MODM is presented. The NIS is the one that

$$\text{Min } f_j(\underline{x}), j \in J$$

$$\text{Max } f_j(\underline{x}), i \in I$$

s.t.

$$\underline{x} \in X$$

An negative ideal solution can be defined as $A^- = (f_1^-, f_2^-, \dots, f_k^-)$, where f_1^- is a feasible and worst value for the 1th objective function. The NIS payoff table is given as:

	f_1	f_2	.	.	.	f_k	
f_1	f_1^-	z_{12}	.	.	.	z_{1k}	(diagonal elements are the NIS)
f_2	z_{21}	f_2^-	.	.	.	z_{2k}	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
.	.	.	.	.	.	.	
f_k	z_{k1}	z_{k2}	.	.	.	f_k^-	

The PIS and NIS tables should be presented before applying any MODM approaches.

Preferred Solution:

A preferred solution is a nondominated solution which is chosen by the DM, through some additional criteria, as the final decision.

Satisficing Solution:

A satisficing solution is a reduced subset of the feasible set, which exceeds all of the aspiration levels (goals) of each objective and needs not be nondominated.

3.4 Interactive Sequential Goal Programming II (ISGP II)

ISGP II [12] is an updated ISGP of Masud and Hwang [21] which combines the attractive features of other MODM approaches with a normalization process of positive ideal and negative ideal solution. The Maximum Achievable Rate (MAR) and the Maximum Achievable Goal (MAG) are introduced. The MAG is a set of nondominated solutions which is the individual DM's largest amount of resources he / she can have without the sacrifices of the other DMs. MAR is the maximum percentage of resources a DM can have without jeopardizing the group harmony.

In the decision processes of ISGP II, first we get the MAG and MAR, a set of nondominated solution, then, given a set of goals, procedures of trade-off between objectives are carried out until achieving DM's maximum satisfactions.

3.5 Illustration

In this section, a bank balance-sheet management problem, using maturity gap approach and numerical data as specified in Sealey [33], Eatman and Sealey [17] and Tayi and Leonard [37], is presented. This example incorporates ISGP II and decision maker's trade-offs between competing and conflicting objectives.

3.5.1 Problem Description

Figure 3-3 shows the basic financial data for the hypothetical bank. The bank has total assets of \$71 million invested in cash, federal funds sold (funds lent to other banks), US Treasury and municipal (state and local government) securities, and loans. The bank has acquired funds through demand and savings deposits, negotiable certificates of deposit or CDs (large denomination time deposits), federal funds purchased (funds borrowed from other banks), Federal Reserve borrowings and equity capital. The total amount of funds available during the period is \$12.2 million plus additional borrowings (difference between \$18.2 million in maturing assets and \$6 million in maturing obligations). The decision variables of x_1 to x_{10} represent possible uses of funds, variables of x_{11} to x_{13} represent possible sources of additional funds. The auxiliary variables of x_{14} to x_{16} are the result of the piecewise linear formulation of the capital-adequacy

	Beginning balances	Amount maturing	x_1
Assets			
Cash	\$6.4	\$6.4	x_1
Federal funds sold	1.2	1.2	x_2
Short-term securities	3.6	3.6	x_3
Government securities, 1-5 years	3.5	0.2	x_4
Government securities, 5-10 years	3.1	0.8	x_5
Government securities, over 10 years	9.0	0.9	x_6
Long-term municipal bonds	7.6	0.5	x_7
Commercial loans	15.5	2.0	x_8
Mortgage loans	9.0	0.9	x_9
Instalment loans	12.1	1.7	x_{10}
	-----	-----	
Total assets	\$71.0	\$18.2	
Liabilities and capital			
Demand deposits	40.0	---	
Savings deposits	20.0	---	
Federal funds purchased	1.0	1.0	x_{11}
Federal Reserve borrowings	2.0	2.0	x_{12}
Negotiable CDs	3.0	3.0	x_{13}
Capital	5.0	---	
	-----	-----	
Total liabilities and capital	\$71.0	\$6.0	

Figure 3-3 Bank Balance Sheet (\$ in million)

objective. The objectives of this problem are to maximize net after-tax profits and to minimize the capital adequacy and risk assets given in equation (3.1), (3.2) and (3.3) respectively. The coefficients of profit objective, (3.1) are the after-tax profit or cost associated with each decision variable.

The capital-adequacy objective, (3.2), assumes that a bank's need for capital is related to the composition of its assets and liabilities. Assets with greater risk and less liquidity are assumed to require more capital backing, the coefficients on the decision variables reflect the relative need for capital: a higher coefficient indicates greater risk and / or less liquidity and thus a greater capital requirement.

The risk-asset objective, (3.3), considered municipal bonds (x_7), commercial loans (x_8), mortgage loans (x_9) and instalment loans (x_{10}) as " risk " assets. The coefficients on the decision variables reflect the relative riskiness of these assets; instalment loans are assumed to have the highest level of credit and interest-rate risk, while commercial loans are assumed to have the lowest level of risk.

The environmental and policy constraints are presented in equations (3.4) to (3.15). The balance-sheet constraint (3.4) requires that sources of funds (\$12.2 millions plus additional borrowings) equal uses of funds (

composition of assets). The reserve requirement constraint (3.5) assumes reserve requirements of 14% on demand deposits (40 millions) and 4% on savings deposits (20 millions) and negotiable CDs. Constraints (3.6) to (3.8) and auxiliary variables x_{14} , x_{15} and x_{16} reflect the specific nature of the capital-adequacy objective. Appendix A develop these constraints and auxiliary variables in detail. Constraint (3.9) requires that federal funds sold be no less than 40% of federal funds purchased. Short-term securities are constrained to be at least 4% of demand and savings deposits and negotiable CDs as in (3.10). Constraints (3.11) to (3.14) confine total borrowing to 10% of the deposits plus capital and require that no single source can comprise more than 60% of the maximum. The constraint (3.15) requires that commercial loans be at least 25% of available funds.

The formulation of this multiple objective linear programming problem is as follows:

$$\begin{aligned}
 (3.1) \quad \text{Max} \quad f_1(\underline{x}) = & .052 x_2 + .053 x_3 + .056 x_4 + .058 x_5 + \\
 & .059 x_6 + .062 x_7 + .076 x_8 + .071 x_9 + \\
 & .095 x_{10} - .052 x_{11} - .05 x_{12} - \\
 & .055 x_{13}
 \end{aligned}$$

$$\begin{aligned}
 (3.2) \quad \text{Min} \quad f_2(\underline{x}) = & .001 x_2 + .001 x_3 + .008 x_4 + .008 x_5 + \\
 & .012 x_6 + .02 x_7 + .02 x_8 + .02 x_9 + \\
 & .02 x_{10} + .013 x_{14} + .008 x_{15} + .019 x_{16}
 \end{aligned}$$

$$(3.3) \quad \text{Min } f_3(\underline{x}) = 1.2 x_7 + 1.0 x_8 + 1.2 x_9 + 1.4 x_{10} \\ \text{s.t.}$$

$$(3.4) \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \\ - x_{11} - x_{12} - x_{13} = 12.2$$

$$(3.5) \quad x_1 - .04 x_{13} \geq 6.4$$

$$(3.6) \quad x_1 + .995 x_2 + .995 x_3 + .96 x_4 - x_{11} - x_{12} - x_{13} \\ + x_{14} \geq 22.832$$

$$(3.7) \quad x_1 + .995 x_2 + .995 x_3 + .96 x_4 + .9 x_5 - x_{11} - \\ x_{12} - x_{13} + x_{15} \geq 20.762$$

$$(3.8) \quad x_1 + .995 x_2 + .995 x_3 + .96 x_4 + .9 x_5 + .85 x_6 \\ - x_{11} - x_{12} - x_{13} + x_{16} \geq 13.877$$

$$(3.9) \quad x_2 - .4 x_{11} \geq 0$$

$$(3.10) \quad x_3 - .04 x_{13} \geq 2.4$$

$$(3.11) \quad x_{11} + x_{12} + x_{13} \leq 6.5$$

$$(3.12) \quad x_{11} \leq 3.9$$

$$(3.13) \quad x_{12} \leq 3.9$$

$$(3.14) \quad x_{13} \leq 3.9$$

$$(3.15) \quad x_8 - .25 x_{11} - .25 x_{12} - .25 x_{13} \geq 3.05$$

$$x_i \geq 0, \quad i = 1, \dots, 16$$

3.5.2 Decision Process With ISGP II

Let constraints be denoted by $\underline{x} \in X$. The problem is solved by applying ISGP II as shown in following steps.

Step 0. Obtain and Construct PIS & NIS Payoff Table.

We solve $\text{Max } f_1(\underline{x})$, $\text{Min } f_2(\underline{x})$ and $\text{Min } f_3(\underline{x})$ subject to

the constraints, respectively, and obtain the PIS-Payoff Table as :

	f_1	f_2	f_3
Max $f_1(\underline{x})$	.627 [*]	.823	11.699
Min $f_2(\underline{x})$	.359	.424 [*]	3.05
Min $f_3(\underline{x})$	.377	.425	3.05 [*]

$$\underline{f}^* = (.627, .424, 3.05)$$

Similarly, we solve Min $f_1(\underline{x})$, Max $f_2(\underline{x})$ and Max $f_3(\underline{x})$ subject to $\underline{x} \in X$, respectively, and obtain the NIP-Payoff Table as:

	f_1	f_2	f_3
Min $f_1(\underline{x})$	.146 ⁻	.522	4.675
Max $f_2(\underline{x})$	.627	.823 ⁻	11.699
Max $f_3(\underline{x})$	.627	.823	11.699 ⁻

$$\underline{f}^- = (.146, .823, 11.699)$$

Step 1.1 obtain a set of MAG and MAR

We (the analysts) set $B\% = 75\%$ and solve the following problem:

$$\text{Min } a^g = \{ d_1^- + d_2^- + d_3^- \} \quad (3.16)$$

s.t.

$$\underline{x} \in X$$

$$f_1(\underline{x}) + .481 d_1^- - .481 d_1^+ = b_1 = .507$$

$$f_2(\underline{x}) - .399 d_2^- + .399 d_2^+ = b_2 = .524$$

$$f_3(\underline{x}) - 8.649 d_3^- + 8.649 d_3^+ = b_3 = 5.212$$

The solution is given as:

$$\underline{x} = (6.400, 0, 2.400, 0.097, 0, 0, 0, 3.398, 0, \\ 1.296, 0, 1.390, 0, 15.341, 13.271, 6.384)$$

$$\underline{f}^g = (.444, .524, 5.212)$$

$$a^g = .130 \geq 0$$

The answer $a^g \geq 0$ implies that with the given $\underline{b}$ there exists a nondominated $\underline{f}^g$. Since $B\% = 75\%$ yields to a nondominated $\underline{b}$ and $\underline{f}^g$, we decrease $B\%$ by 5%, i.e. $B\% = 70\%$ and recalculate (3.16) with:

$$b_{70} = (.483, .544, 5.645)$$

and obtaining

$$\underline{x} = (6.400, 0, 2.400, .098, 0, 0, 0, 3.381, 0,$$

1.546, 0, 1.725, 0, 15.675, 13.605, 6.720)

$$\underline{f}^g = (.458, .544, 5.645)$$

$$a^g \geq 0$$

Again, we find the new $a^g \geq 0$ and $\underline{f}^g$ a set of nondominated solution. We repeated the procedure by increasing (if $a^g \geq 0$) or decreasing (if $a^g \leq 0$) B% until a minimum nonzero a^g is obtained. This implies that $\underline{f}^g$ is a set of nondominated solution which is the MAG we desire. For this problem, the MAR is 67% and the solution of MAG is:

$$\underline{x} = (6.400, 0, 2.400, .101, 0, 0, 0, 3.532, 0, \\ 1.694, 0, 1.927, 0, 15.874, 13.804, 6.919)$$

$$\underline{f}^g = (.466, .556, 5.904)$$

$$\underline{F}^g = (4.291, 1.480, 52.204)$$

where $\underline{F}^g$ is referred to total profits, capital adequacy ratio and risk asset ratio with respect to total assets and liabilities.

Step 1.2 Display ISGP II Initial Table

The initial table is presented to the DM.

	Max $f_1(\underline{x})$	Min $f_2(\underline{x})$	Min $f_3(\underline{x})$
$\underline{f}^*$	.627	.424	3.050
MAG 62%	.466	.556	5.904
$\underline{f}^-$	.146	.823	11.699
$\underline{w}$	1.000	1.000	1.000

We assume the DM does not accept the MAG as the preferred solution, then we go through the trade-off procedure to obtain a preferred solution which is still a nondominated solution.

Step 1.3 Set Initial Goals $\underline{b}$.

The DM set his / her initial goals at $\underline{b}$ referring to the initial table. The initial goals, $\underline{b}$, should satisfy

$$f_j < b_j \leq f_j^*, j \in J$$

$$f_i^* \leq b_i < f_i, i \in I$$

Here we assume $\underline{b}$ at $B\% = 70\%$ as

$$\underline{b} = (.483, .544, 5.645)$$

Step 2.1 Solve for PS (Principal Solution)

The problem is to find $\underline{x}$ so as to

$$\text{Min } a_{ps} = \{ d_1^- + d_2^- + d_3^- \} \quad (3.17)$$

s.t.

$$\underline{x} \in X$$

$$f_1(\underline{x}) + .481 d_1^- - .481 d_1^+ = b_1 = .483$$

$$f_2(\underline{x}) - .399 d_2^- + .399 d_2^+ = b_2 = .544$$

$$f_3(\underline{x}) - 8.649 d_3^- + 8.649 d_3^+ = b_3 = 5.645$$

$$0 \leq d_j \leq 1, \quad j = 1, 2, 3$$

and the solution is

$$\underline{x} = (6.400, 0, 2.400, .098, 0, 0, 0, 3.381, 0, \\ 1.546, 0, 1.725, 0, 15.675, 13.605, 6.720)$$

$$\underline{f}^0 = (.458, .544, 5.645)$$

$$a_{ps} \geq 0$$

Step 2.2

As a_{ps} implies that the solution $\underline{f}^0$ is a nondominated solution and is accepted as the principal solution

Step 3 Solve for ASs (Auxiliary Solutions)

For all three objectives, we need to solve their auxiliary solutions respectively:

$$\text{Min } a^1 = \{ d_2^- + d_3^- \} \quad (3.18)$$

s.t.

$$\underline{x} \in X$$

$$f_1(\underline{x}) \geq b_1 = .483$$

$$f_2(\underline{x}) - .399 d_2^- + .399 d_2^+ = b_2 = .544$$

$$f_3(\underline{x}) - 8.649 d_3^- + 8.649 d_3^+ = b_3 = 5.645$$

$$0 \leq d_j \leq 1, \quad j = 2, 3$$

where $a^1 \geq 0$,

$$\text{Min } a^2 = \{ d_1^- + d_3^- \} \quad (3.19)$$

s.t.

$$\underline{x} \in X$$

$$f_1(\underline{x}) + .481 d_1^- - .481 d_1^+ = b_1 = .483$$

$$f_2(\underline{x}) \leq b_2 = .544$$

$$f_3(\underline{x}) - 8.649 d_3^- + 8.649 d_3^+ = b_3 = 5.645$$

$$0 \leq d_j \leq 1, \quad j = 1, 3$$

where $a^2 \geq 0$,

$$\text{Min } a^3 = \{ d_1^- + d_2^- \} \quad (3.20)$$

s.t.

$$\underline{x} \in X$$

$$f_1(\underline{x}) + .481 d_1^- - .481 d_1^+ = b_1 = .483$$

$$f_2(\underline{x}) - .399 d_2^- + .399 d_2^+ = b_2 = .544$$

$$f_3(\underline{x}) \leq b_3 = 5.645$$

$$0 \leq d_j \leq 1, \quad j = 1, 2$$

Step 4 Formulating ISGP II Trade-off Table

	Max $f_1(\underline{x})$	Min $f_2(\underline{x})$	Min $f_3(\underline{x})$
$\underline{f}^*$	.627	.424	3.050
MAG 67%	.569	.452	3.483
$\underline{f}^-$	.146	.823	11.699
$\underline{w}$	1.000	1.000	1.000
Iteration			
Goal	.483	.544	5.645
PS	.458	.544	5.645
AS1	.483	.581	6.475
AS2	.458	.544	5.645
AS3	.458	.544	5.645

This trade-off table is presented to DM for evaluation. If the DM is satisfied with this set of solution the algorithm terminates here, if not, given a new set of $\underline{b}$ and start another iteration.

CHAPTER 4

FUZZY MULTIPLE OBJECTIVE PROGRAMMING MODEL

The inherent uncertainty of a bank's deposit flows, loan demand, cost of funds and return on investment (interest rate), along with the increased variability of economic conditions have emphasized the need for greater efficiency in managing bank balance-sheet. Yet, this uncertainty is inadequately handled by the existing models.

In this chapter, we develop a model which incorporates the contemporary fuzzy set theory and integrates the bank balance-sheet management approaches, forecasting, simulation and mathematical programming, we have discussed in previous chapters. For the simplicity of the problem, only the fuzziness on the coefficients of the objectives will be discussed.

4.1 Introduction to Fuzzy Sets

In this section some concepts of fuzzy set theory are introduced before incorporating to bank model. They are fuzziness, possibility, crisp set, fuzzy set and α -level set.

4.1.1 Fuzziness

Due to lack of information the future state of the system might not be known completely. This type of uncertainty (stochastic character) has long been handled appropriately by probability theory and statistics. Another type of uncertainty concerning the description of the semantic meaning of the events, phenomena or statements shall be called fuzziness [42].

Fuzziness can be found in many areas of daily life, such as in engineering, in medicine, in meteorology, in manufacturing and other areas in which human judgment, evaluation and decisions are important. There are the areas of decision making, reasoning, learning, and so on. Others are most of our daily communication uses "natural languages" and a good part of our thinking. Even the words we used are well defined, the using for labelling a set the boundaries usually become fuzzy or vague. Examples are words such as "tall men", "beautiful women", or the financial terms such as "creditworthy customers", "profitable assets", "liquidity", and so on.

In 1965, Zadeh [40] proposed "Fuzzy Sets Theory" which

was intended to improve the oversimplified model with a more complex and flexible model in order to resolve the real world systems involving human aspects. It helps the decision maker not only to consider the existing alternatives under given constraints -- to optimize a given system, but to develop new alternatives -- design a system [42].

4.1.2 Crisp Set

Before introducing the concept of fuzzy set, let us first look at crisp set.

A classical (crisp) set is normally defined as a collection of elements or objects $x \in X$. Each single element can either belong to or not belong to a set A , $A \subseteq X$. Here is an example: a crisp set $A = \{ x \mid x = 2y, y \in N \}$. That is, A is a set of all even natural numbers. This crisp set A can then represented as:

$$A = \{ (1, 0), (2, 1), (3, 0), (4, 1), \dots \}$$

where the first number of order pairs, 1, 2, 3, 4, ..., represents the element of the set and the second number is the measure of membership. This membership in a crisp subset, A , of X is often viewed as characteristic function from X to $\{ 0, 1 \}$ such that

$$\begin{aligned}\mu_A(x) &= 1 && \text{if and only if } x \in A \\ &= 0 && \text{if and only if } x \notin A\end{aligned}$$

where $\{ 0, 1 \}$ is called a valuation set.

4.1.3 Fuzzy Set

Fuzzy set is a set with a vague, cloudy or fuzzy boundary, the characteristic function allows various degrees of membership for the elements of a given set. If the valuation set is allowed to be the real interval $[0, 1]$, then, $\tilde{A}$ is called a fuzzy set. $\mu_{\tilde{A}}(x)$ is the degree of membership of x in $\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(x)$ is to 1, the more x belongs to $\tilde{A}$. $\tilde{A}$ can be depicted as

$$\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) \mid x \in X \}$$

where the characteristic function, $\mu_{\tilde{A}}(x)$, can also be called as a possibility distribution or a membership function.

Consider another example [39]: A realtor wants to classify the houses he offers to his clients. One indicator of comfort of these houses is the number of bedrooms in it. Let $X = \{ 1, 2, 3, 4, \dots, 8 \}$ be the set of available types of houses described by x = number of bedrooms in a house. Then the fuzzy set "comfortable type of house for a 4-person family" may be described as

$$\tilde{A} = \{ (1, 0), (2, .3), (3, .7), (4, 1), (5, 1), (6, .7), (7, .3), (8, .0) \}$$

Figure 4-1 shows the derived degree of membership for the fuzzy set $\tilde{A}$.

4.1.4 α -Level Set

Definition: The (crisp) set of elements which belong to the fuzzy set $\tilde{A}$ at least to the degree α is called the α -level set:

$$A_{\alpha} = \{ x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha \}$$

$$A_{\alpha}' = \{ x \in X \mid \mu_{\tilde{A}}(x) > \alpha \}$$

where A_{α}' is called "strong α -level set" or "strong α -cut".

Here we use the same example and list possible α -level sets:

$$A_{.3} = \{ 2, 3, 4, 5, 6, 7 \}$$

$$A_{.7} = \{ 3, 4, 5, 6 \}$$

$$A_1 = \{ 5, 6 \}$$

and the strong α -level set for $\alpha = .7$ is

$$A_{.7}' = \{ 5, 6 \}$$

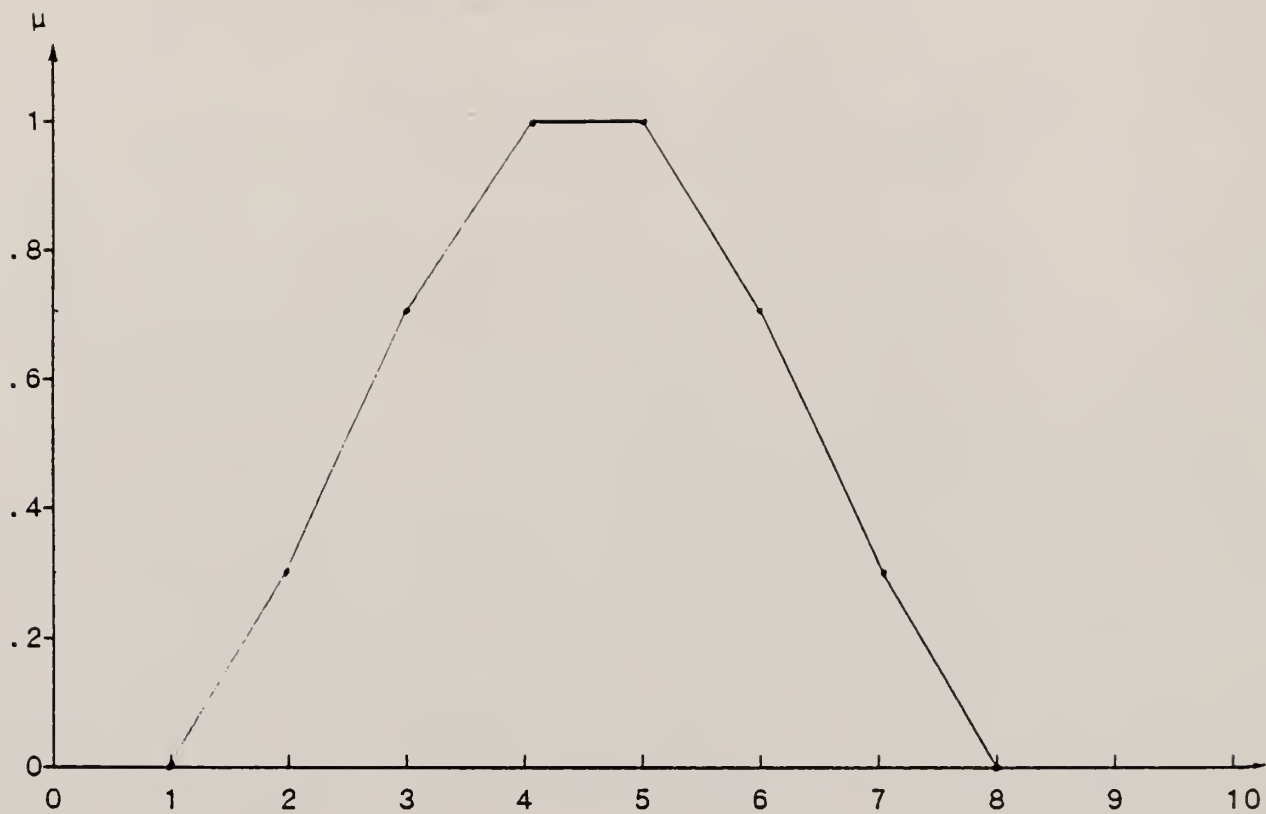


Figure 4-1 Degree of Membership Function of Fuzzy Set A

4.2 Fuzzy Numbers and Triangular Fuzzy Numbers

In this section we will show the definition and examples of fuzzy numbers. One type of fuzzy numbers called triangular fuzzy numbers which will be used in our model are also presented.

4.2.1 Fuzzy Numbers

Definition: A fuzzy number $\tilde{M}$ is a convex normalized fuzzy set $\tilde{M}$ of the real line R such that

1. It exists exactly one $x_0 \in R$, $\mu_{\tilde{M}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{M}$)
2. $\mu_{\tilde{M}}(x)$ is piecewise continuous.

Definition: A fuzzy number $\tilde{M}$ is called positive (negative) if its membership function is such that $\mu_{\tilde{M}}(x) = 0$, $\forall x < 0$ ($\forall x > 0$).

Here are some examples for explaining fuzzy numbers:

Approximately

$$\begin{aligned} 5 &= \{ (3, .2), (4, .6), (5, 1), (6, .7), \\ &\quad (7, .1) \} \\ 10 &= \{ (8, .3), (9, .7), (10, 1), (11, .7), \\ &\quad (12, .3) \} \end{aligned}$$

The formulation of fuzzy linear programming incorporating fuzzy numbers is as follows:

$$\text{Max } \tilde{c}^T x \quad (4.1)$$

s.t.

$$(\tilde{A} x)_i \leq \tilde{b}_i$$

$$x \geq 0 \quad \forall i$$

where the fuzzy objective function is characterized by its membership function, and so are the constraints.

There are many approaches [28] available for solving fuzzy linear programming problems, all these approaches transform the fuzzy programs to crisp ones and solve the problems by traditional linear programming.

4.2.2 Triangular Fuzzy Numbers

Tanaka et al. [36] considered the problem given by equation 4.1 as:

$$\text{find } x \quad (4.2)$$

such that

$$\tilde{Y}_0 = -\tilde{B}_0 + \tilde{A}_{01} x_1 + \dots + \tilde{A}_{0n} x_n \geq 0$$

$$\tilde{Y}_1 = \tilde{B}_1 - (\tilde{A}_{11} x_1 + \dots + \tilde{A}_{1n} x_n) \geq 0$$

$$\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}$$

$$\widetilde{Y}_m = \widetilde{B}_m - (\widetilde{A}_{m1} x_1 + \dots + \widetilde{A}_{mn} x_n) \geq 0$$

where $\widetilde{B}_0 = (b_0, p_0)$, $\widetilde{B}_1 = (b_1, p_1)$, $\widetilde{A}_{0j} = c_j = (a_{0j}, d_{0j})$ and $A_{ij} = (a_{ij}, d_{ij})$, $\forall i, j$. The first numbers of the ordered pairs are the centers of the triangular membership functions and the second numbers expressing the fuzziness are the deviations from the corresponding centers. These triangular membership functions $\mu_{\widetilde{B}_i}$ and $\mu_{\widetilde{A}_{ij}}$, $\forall i, j$, of the fuzzy parameters are described as follows (see Figure 4-2a, 4-2b):

$$\mu_{\widetilde{B}_i}(B_i) = \begin{cases} 1 - \frac{|B_i - b_i|}{p_i} & \text{if } b_i - p_i \leq B_i \leq b_i + p_i \\ 0 & \text{otherwise,} \end{cases}$$

$$\mu_{\widetilde{A}_{ij}}(A_{ij}) = \begin{cases} 1 - \frac{|A_{ij} - a_{ij}|}{d_{ij}} & \text{if } a_{ij} - d_{ij} \leq A_{ij} \leq a_{ij} + d_{ij} \\ 0 & \text{otherwise,} \end{cases}$$

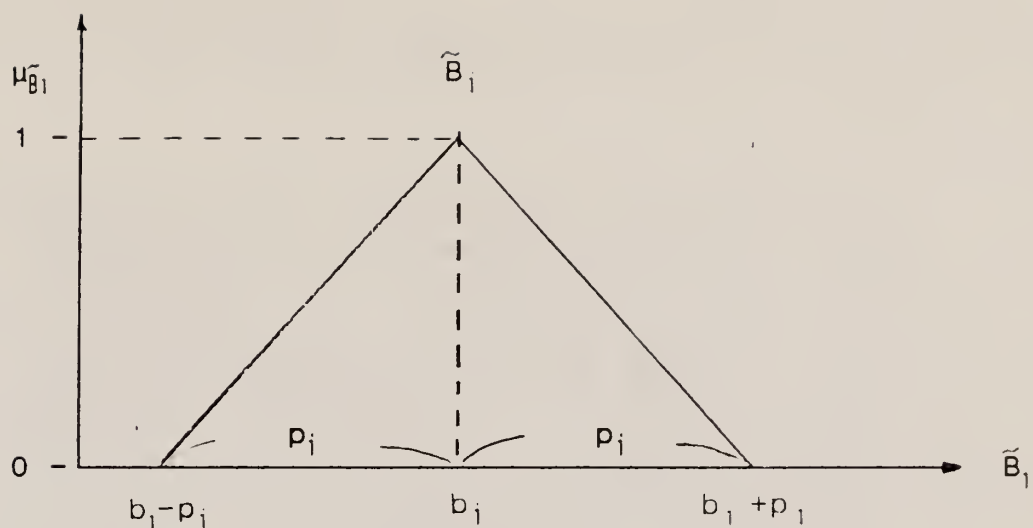


Figure 4-1 The membership functions of $\tilde{B}_i$, $i = 1, \dots, m$

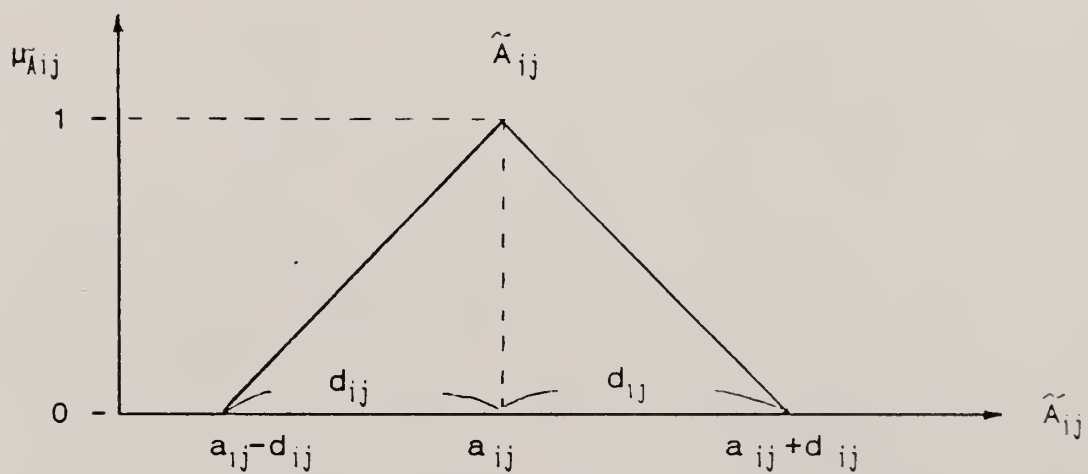


Figure 4-2 The membership functions of $\tilde{A}_{ij}$, $i = 1, \dots, m$ and $j = 1, \dots, n$

4.3 Fuzzy Multiple Objective Decision Making

In a deterministic model, an positive or negative ideal solution is exactly defined. While dealing with inexact information or fuzzy cognitive and decision making processes, it may be impossible to identify the ideal solution with exactitude. Leung [26] employed concepts of the theory of possibility and provides a formal conceptualization of a fuzzy ideal. It shows that if the ideal solution is a region with fuzzy boundary, then the corresponding compromise solution set becomes imprecise. Different compromise solution may be obtained depending on which element of the fuzzy ideal is selected as a reference. Comparing to the objective function space representation of two objectives of a deterministic model depicted in Figure 3-3, we present the fuzzy positive ideal and fuzzy negative ideal solutions through an objective function space representation of two objectives (see Figure 4-3). The numerical example of fuzzy positive ideal and fuzzy negative ideal solutions will be shown in section 4.4.2.

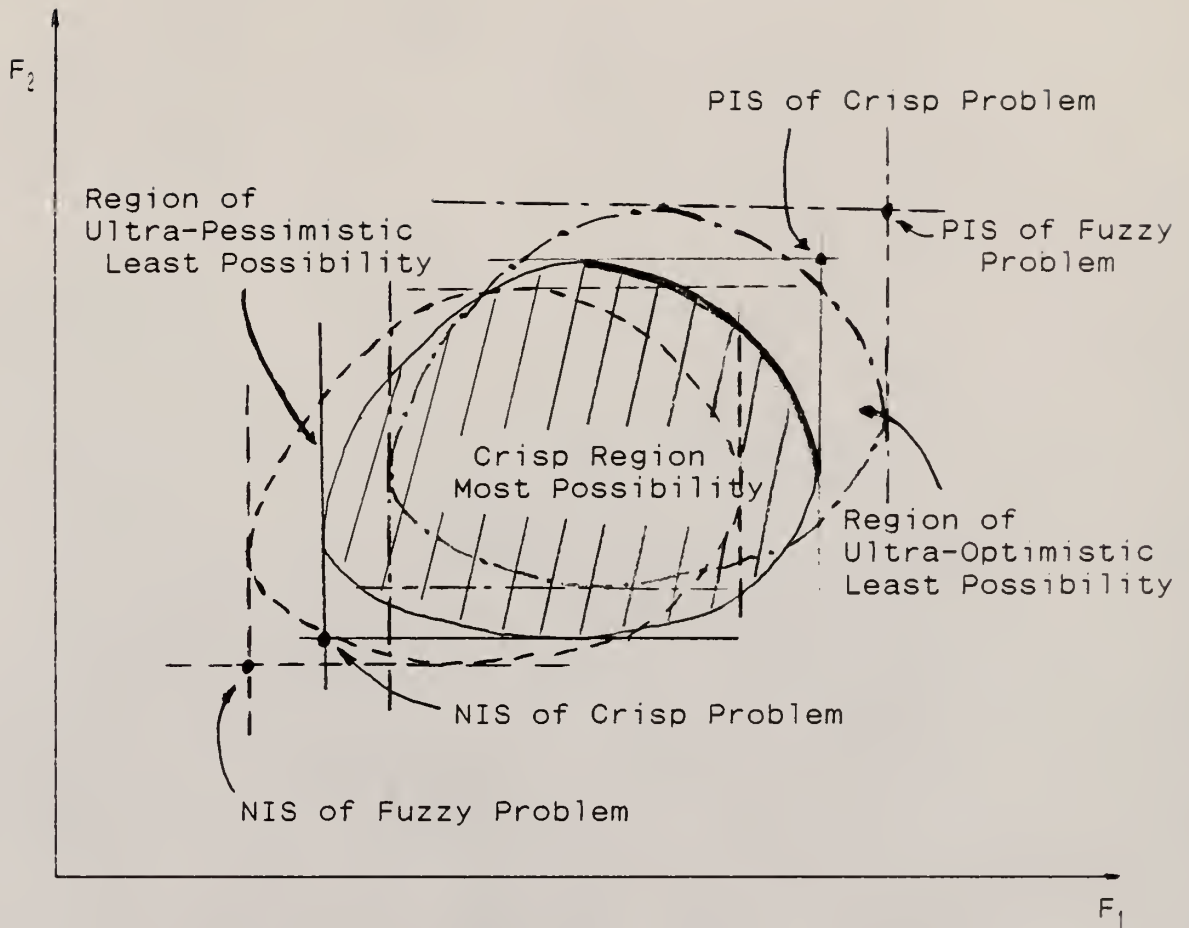


Figure 4-3 Fuzzy Objective Function Space Representation of Two Objectives

4.4 Fuzziness in Bank Balance-Sheet Management Models

Referring to the example shown in chapter 3, we may consider the fuzziness in the coefficients of objective functions: the after-tax profit or the cost of fund, the relative need for capital and the relative riskiness of the assets are uncertain under economic conditions and subject to decision makers' subjective judgments; we may consider the fuzziness in resources: the cash flows are uncertain due to customers' behaviors; we may also consider the fuzziness in technical coefficients of the constraints: the regulations set up by regulators and the policies chosen by senior executives are uncertain. These uncertain situations are not well handled by existing approaches. For the simplicity of the problem, we only consider the fuzziness in objective functions and use triangular fuzzy numbers in our model. Moreover, in the further study we may focus on fuzzifying both objectives and constraints. As the approach used in Chapter 3, we still apply the ISGP II to obtain the MAR and MAG for this fuzzy MODM problem.

4.4.1 Fuzziness in the Coefficients of Objective Functions

In our model, we consider that it is reasonable to define the coefficients of objective functions as non-symmetric triangular fuzzy numbers because of the economic uncertain nature and decision makers' subjective judgments.

Figure 4-4 shows a possibility distribution of

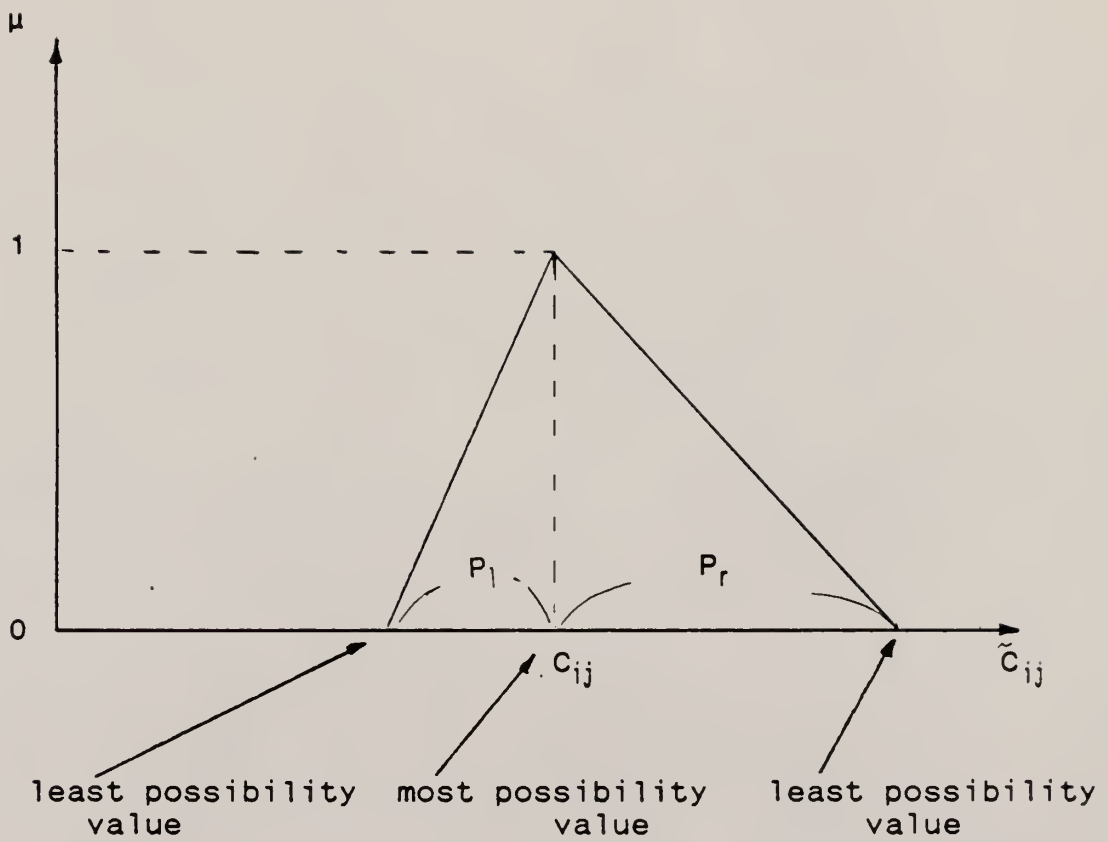
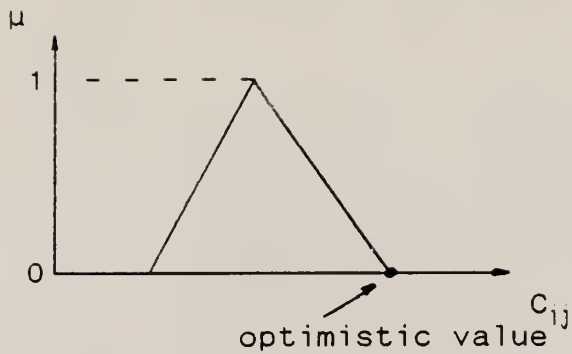
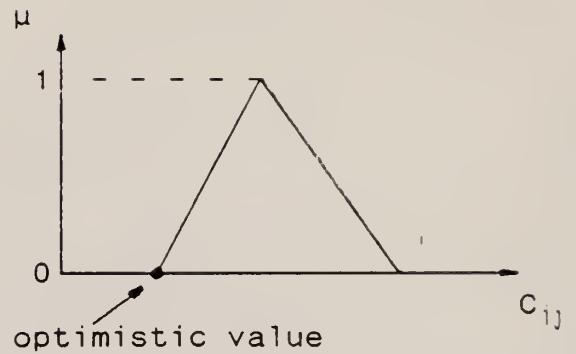


Figure 4-4 Possibility distribution of Triangular Fuzzy Numbers

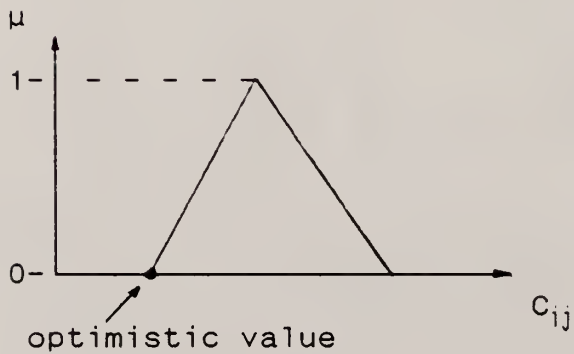
triangular fuzzy numbers. At point C_{ij} in Figure 4-4, it represents the most possibility value of coefficient with the membership function $\mu_{C_{ij}} = 1$. Sometimes, the membership function will be less than 1. That means the possibility is less than 1. As the deviation from the most possibility value increases, the distribution of possibility decreases linearly. The notations P_r and P_l represent the deviation from most possibility value to right boundary (upper bound) and to left boundary (lower bound). Both boundaries are the least possibility value of coefficient with membership function value $\mu = 0$. The bases of triangulars are the sum of deviations, P_r and P_l . These deviations may be obtained through empirical analysis of previous data and from DMs' judgment. In our model, we assume that $P_r : P_l = 2 : 1$ for all coefficients in three objectives. The intervals sum up from the deviations are considered as the DMs' acceptable coefficient values. Therefore, within these intervals, for a maximizing objective, the best values of coefficients can be obtained at the upper bounds of the range (ultra-optimistic) if they are positive (see Figure 4-5a) or at the lower bounds of the range (ultra-pessimistic) if they are negative (see Figure 4-5b). For a minimizing objective, the best values of coefficients exist at the lower bounds of the range if they are positive (see Figure 4-5c) or at the upper bounds of the range if they are negative (see Figure 4-5d). The worst values of



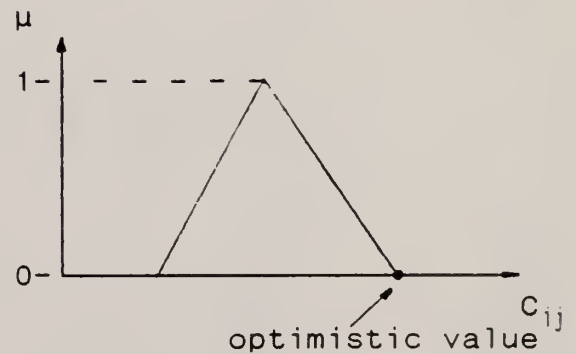
(a) maximizing with positive coefficients



(b) maximizing with negative coefficients



(c) minimizing with positive coefficients



(d) minimizing with negative coefficients

Figure 4-5 Upper and Lower Bounds of Fuzzy Intervals

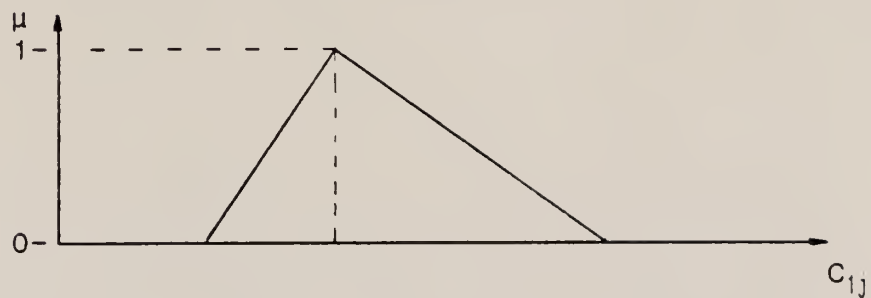
coefficients exist at reversed bounds of each of maximizing objective and minimizing objective.

We may interpret the fuzziness of coefficients of objective functions of bank model in terms of real-world situations. For all sources and uses in profit objective, the returns on investment (cost of fund), the interest rates, have the least possibility of rising 2% or have the least possibility of falling 1% with respect to most possibility interest rate. Similarly, the coefficients on the decision variables of capital adequacy objective which measure the relative need for capital and the coefficients of risk asset ratio objective which reflect the relative riskiness of these assets may be fuzzy subject to decision makers' intuitive judgments. The upper and lower bounds of these triangular fuzzy numbers are depicted in Figure 4-6a , 4-6b and 4-6c.

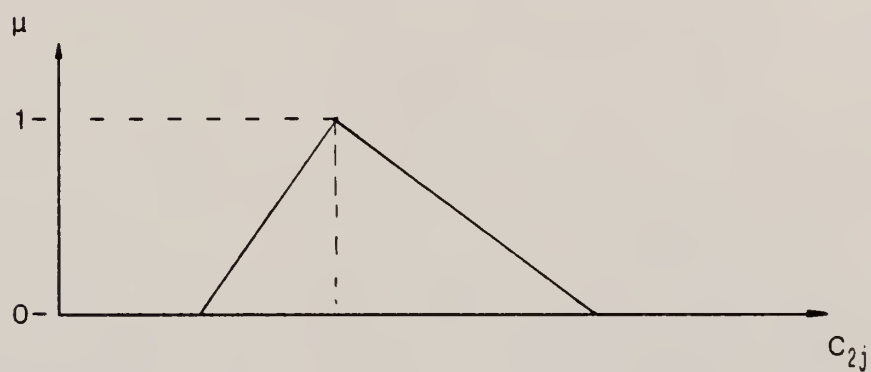
The formulation of this fuzzy multiple objective model is as follows:

$$\begin{aligned}
 (4.3) \quad & \text{Max} \quad \tilde{C}_1^T \underline{x} \\
 & \text{Min} \quad \tilde{C}_2^T \underline{x} \\
 & \text{Min} \quad \tilde{C}_3^T \underline{x} \\
 & \text{s.t.} \\
 & \quad \underline{x} \in X \\
 & \quad \underline{x} \geq 0
 \end{aligned}$$

(a) Profit Objective



(b) Capital Adequacy Objective



(c) Risk Asset Ratio Objective

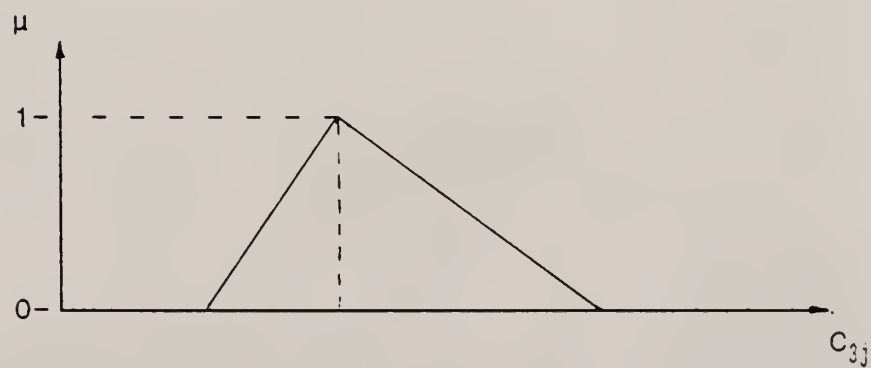


Figure 4-6 Fuzzy Interval of Coefficients of Three Objectives

where x are the decision variables and X are the constraints of the model.

The the fuzziness of the coefficients of the objective functions are listed in Table 4-1. The notations used in Table 4-1 and the subsequent tables as "c1", "c2" and "c3" are the coefficient of each objective and decision variable, "u" and "l" stand for upper bound and lower bound respectively.

4.4.2 Fuzzy version of ISGP II

Again, we apply ISGP II to obtain MAR and MAG in this fuzzy multiple objective model. In most of the fuzzy linear programming problems, people always use operators to solve the problems, here only the concept of α -level set is employed as it is considered most suitable for our model.

In concept, the value α of α -level set may be considered as a minimum subjectively acceptable possibility (which is an analog of the least acceptable reliability) to all triangular fuzzy numbers. Those situations whose membership functions are below α are considered unacceptable in DMs' point of view. For example, let us assume $\alpha = 0.5$. We can obtain an interval of the acceptable coefficient values with their membership value larger than or equal to 0.5.

Now let us consider the previous MODM model with fuzzy Coefficients in objectives. Table 4-2 shows the upper and

Table 4-1 Fuzziness of the coefficients of the objective functions

	Crisp coefficients			Fuzzy coefficients					
	c1	c2	c3	clu	c1l	c2u	c2l	c3u	c3l
x1	0.000	0.0000	0.00	0.0000	0.0000	0.0000	0.0000	0.000	0.000
x2	0.052	0.0010	0.00	0.0720	0.0420	0.0030	0.0000	0.000	0.000
x3	0.053	0.0010	0.00	0.0730	0.0430	0.0030	0.0000	0.000	0.000
x4	0.056	0.0080	0.00	0.0760	0.0460	0.0120	0.0060	0.000	0.000
x5	0.058	0.0080	0.00	0.0780	0.0480	0.0120	0.0060	0.000	0.000
x6	0.059	0.0120	0.00	0.0790	0.0490	0.0180	0.0090	0.000	0.000
x7	0.062	0.0200	1.20	0.0820	0.0520	0.0300	0.0150	1.400	1.100
x8	0.076	0.0200	1.00	0.0960	0.0660	0.0300	0.0150	1.200	0.900
x9	0.071	0.0200	1.20	0.0910	0.0610	0.0300	0.0150	1.400	1.100
x10	0.095	0.0200	1.40	0.1150	0.0850	0.0300	0.0150	1.600	1.300
x11	-0.052	0.0000	0.00	-0.0420	-0.0720	0.0000	0.0000	0.000	0.000
x12	-0.050	0.0000	0.00	-0.0400	-0.0700	0.0000	0.0000	0.000	0.000
x13	-0.055	0.0000	0.00	-0.0450	-0.0750	0.0000	0.0000	0.000	0.000
x14	0.000	0.0130	0.00	0.0000	0.0000	0.0190	0.0100	0.000	0.000
x15	0.000	0.0080	0.00	0.0000	0.0000	0.0120	0.0060	0.000	0.000
x16	0.000	0.0190	0.00	0.0000	0.0000	0.0290	0.0140	0.000	0.000

Table 4-2 Coefficients of Objective Functions at $\alpha = 0.5$

	Crisp Coefficients			Fuzzy Coefficients					
	c1	c2	c3	clu	c1l	c2u	c2l	c3u	c3l
x1	0.000	0.0000	0.00	0.0000	0.0000	0.0000	0.0000	0.000	0.000
x2	0.052	0.0010	0.00	0.0620	0.0470	0.0020	0.0005	0.000	0.000
x3	0.053	0.0010	0.00	0.0630	0.0480	0.0020	0.0005	0.000	0.000
x4	0.056	0.0080	0.00	0.0660	0.0510	0.0100	0.0070	0.000	0.000
x5	0.058	0.0080	0.00	0.0680	0.0530	0.0100	0.0070	0.000	0.000
x6	0.059	0.0120	0.00	0.0690	0.0540	0.0150	0.0105	0.000	0.000
x7	0.062	0.0200	1.20	0.0720	0.0570	0.0250	0.0175	1.300	1.150
x8	0.076	0.0200	1.00	0.0860	0.0710	0.0250	0.0175	1.100	0.950
x9	0.071	0.0200	1.20	0.0810	0.0660	0.0250	0.0175	1.300	1.150
x10	0.095	0.0200	1.40	0.1050	0.0900	0.0250	0.0175	1.500	1.350
x11	-0.052	0.0000	0.00	-0.0470	-0.0620	0.0000	0.0000	0.000	0.000
x12	-0.050	0.0000	0.00	-0.0450	-0.0600	0.0000	0.0000	0.000	0.000
x13	-0.055	0.0000	0.00	-0.0500	-0.0650	0.0000	0.0000	0.000	0.000
x14	0.000	0.0130	0.00	0.0000	0.0000	0.0160	0.0115	0.000	0.000
x15	0.000	0.0080	0.00	0.0000	0.0000	0.0100	0.0070	0.000	0.000
x16	0.000	0.0190	0.00	0.0000	0.0000	0.0240	0.0165	0.000	0.000

lower bounds of coefficients of each objective of our bank model at $\alpha = 0.5$. By using the concept of α -level set, we can then obtain fuzzy positive ideal solution. The problem becomes (1) to maximize the profit objective where the coefficients are at upper bounds if positive coefficients or at lower bounds if negative coefficients of the ranges stipulated by α -level set; (2) to minimize capital adequacy ratio and risk asset ratio objectives where the coefficients are at lower bounds if they are positive coefficients or at upper bounds if they are negative coefficients.

The formulation of obtaining fuzzy positive ideal solution is shown in equation 4.4 as follows:

$$\begin{aligned}
 (4.4) \quad \text{Max} \quad f_1(\underline{x}) &= .062 x_2 + .063 x_3 + .066 x_4 + .068 x_5 + \\
 &\quad .069 x_6 + .072 x_7 + .086 x_8 + .081 x_9 + \\
 &\quad .105 x_{10} - .047 x_{11} - .045 x_{12} - \\
 &\quad .050 x_{13}. \\
 \text{Min} \quad f_2(\underline{x}) &= .0005 x_2 + .0005 x_3 + .007 x_4 + .007 x_5 \\
 &\quad + .0105 x_6 + .0175 x_7 + .0175 x_8 + \\
 &\quad .0175 x_9 + .0175 x_{10} + .0115 x_{14} + \\
 &\quad .007 x_{15} + .0165 x_{16} \\
 \text{Min} \quad f_3(\underline{x}) &= 1.15 x_7 + .095 x_8 + 1.15 x_9 + 1.35 x_{10} \\
 \text{s.t.} \\
 \underline{x} &\in X
 \end{aligned}$$

where the constraints are unchanged.

It is obvious that the equation of (4.4) is a crisp problem. Therefore, we can obtain the fuzzy PIS-Payoff Table by solving $\text{Max } f_1(\underline{x})$, $\text{Min } f_2(\underline{x})$ and $\text{Min } f_3(\underline{x})$ subject to the constraints, respectively, and obtain the PIS-Payoff Table as :

	f_1	f_2	f_3
$\text{Max } f_1(\underline{x})$	.781 [*]	.720	11.214
$\text{Min } f_2(\underline{x})$	.414	.372 [*]	2.898
$\text{Min } f_3(\underline{x})$	.435	.372	2.898 [*]

$$\underline{f}^* = (.781, .372, 2.898)$$

Similarly, in order to obtain fuzzy negative ideal solution, we reformulate $\text{Min } f_1(\underline{x})$, $\text{Max } f_2(\underline{x})$ and $\text{Max } f_3(\underline{x})$ subject to $\underline{x} \in X$, respectively, as in (4.5):

$$(4.5) \quad \text{Min } f_1(\underline{x}) = .047 x_2 + .048 x_3 + .051 x_4 + .053 x_5 + .054 x_6 + .057 x_7 + .071 x_8 + .066 x_9 + .090 x_{10} - .062 x_{11} - .060 x_{12} - .065 x_{13}$$

$$\text{Max } f_2(\underline{x}) = .002 x_2 + .002 x_3 + .01 x_4 + .01 x_5 + .015 x_6 + .025 x_7 + .025 x_8 + .025 x_9$$

$$\begin{aligned}
& + .025 x_{10} + .016 x_{14} + .01 x_{15} + \\
& .0165 x_{16} \\
\text{Max } f_3(\underline{x}) = & 1.3 x_7 + 1.1 x_8 + 1.3 x_9 + 1.5 x_{10} \\
\text{s.t.} \\
& \underline{x} \in X
\end{aligned}$$

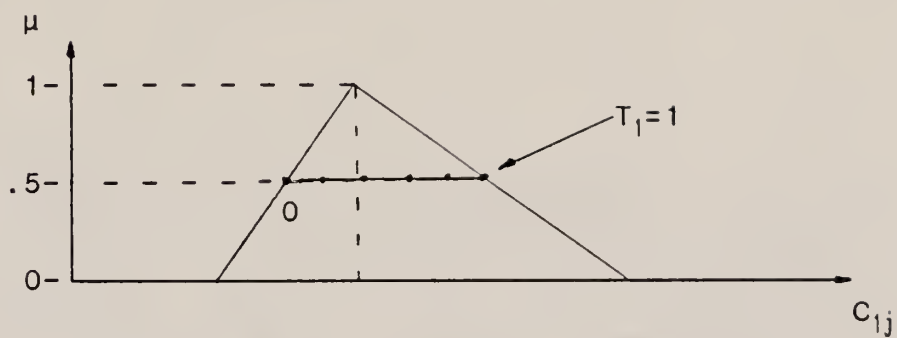
and obtain the fuzzy NIS-Payoff Table as:

	f_1	f_2	f_3
Min $f_1(\underline{x})$	.045 ⁻	.387	5.143
Max $f_2(\underline{x})$	.501	1.029 ⁻	12.688
Max $f_3(\underline{x})$	.501	1.029	12.668 ⁻

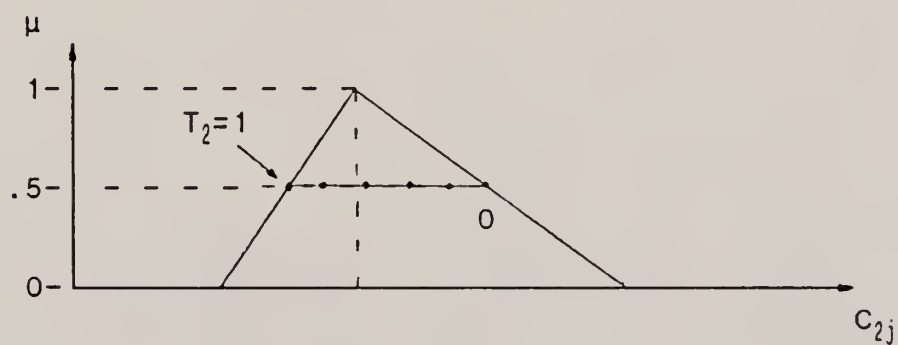
$$\underline{f}^- = (.045, 1.029, 12.668)$$

In order to know the optimal information, the balance-sheet position, of fuzzy bank model, we divide all intervals of coefficients, DMs' acceptable coefficient values, into five equal segments. Figure 4-7 shows the segmentations for three objectives. Considered T_1 , T_2 and T_3 ($T_i = \{ 0, .2, .4, .6, .8, 1 \}$) as weights of intervals for each objective as listed in Table 4-3, the weight T_i in each interval is referred to as the pessimism-optimism index. At $T_i = 1$ is ultra-optimistic and at $T_i = 0$ ultra-pessimistic. Totally,

(a) Profit Objective



(b) Capital Adequacy Objective



(c) Risk Asset Ratio Objective

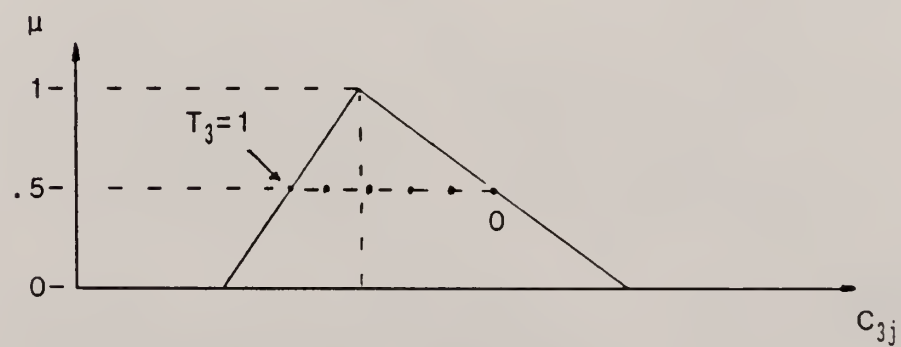


Figure 4-7 α -Level Cut for Coefficients of Three Objectives

Table 4-3 Coefficients of Objectives at Different Weights

T1 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x2	0.0470	0.0500	0.0530	0.0560	0.0590	0.0620
x3	0.0480	0.0510	0.0540	0.0570	0.0600	0.0630
x4	0.0510	0.0540	0.0570	0.0600	0.0630	0.0660
x5	0.0530	0.0560	0.0590	0.0620	0.0650	0.0680
x6	0.0540	0.0570	0.0600	0.0630	0.0660	0.0690
x7	0.0570	0.0600	0.0630	0.0660	0.0690	0.0720
x8	0.0710	0.0740	0.0770	0.0800	0.0830	0.0860
x9	0.0660	0.0690	0.0720	0.0750	0.0780	0.0810
x10	0.0900	0.0930	0.0960	0.0990	0.1020	0.1050
x11	-0.0620	-0.0590	-0.0560	-0.0530	-0.0500	-0.0470
x12	-0.0600	-0.0570	-0.0540	-0.0510	-0.0480	-0.0450
x13	-0.0650	-0.0620	-0.0590	-0.0560	-0.0530	-0.0500
x14	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x16	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
T2 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x2	0.0020	0.0017	0.0014	0.0011	0.0008	0.0005
x3	0.0020	0.0017	0.0014	0.0011	0.0008	0.0005
x4	0.0100	0.0094	0.0088	0.0082	0.0076	0.0070
x5	0.0100	0.0094	0.0088	0.0082	0.0076	0.0070
x6	0.0150	0.0141	0.0132	0.0123	0.0114	0.0105
x7	0.0250	0.0235	0.0220	0.0205	0.0190	0.0175
x8	0.0250	0.0235	0.0220	0.0205	0.0190	0.0175
x9	0.0250	0.0235	0.0220	0.0205	0.0190	0.0175
x10	0.0250	0.0235	0.0220	0.0205	0.0190	0.0175
x11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x13	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
x14	0.0160	0.0151	0.0142	0.0133	0.0124	0.0115
x15	0.0100	0.0094	0.0088	0.0082	0.0076	0.0070
x16	0.0240	0.0225	0.0210	0.0195	0.0180	0.0165
T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	0.000	0.000	0.000	0.000	0.000	0.000
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	0.000	0.000	0.000	0.000	0.000	0.000
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	1.300	1.270	1.240	1.210	1.180	1.150
x8	1.100	1.070	1.040	1.010	0.980	0.950
x9	1.300	1.270	1.240	1.210	1.180	1.150
x10	1.500	1.470	1.440	1.410	1.380	1.350
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	0.000	0.000	0.000	0.000	0.000	0.000
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	0.000	0.000	0.000	0.000	0.000	0.000
x15	0.000	0.000	0.000	0.000	0.000	0.000
x16	0.000	0.000	0.000	0.000	0.000	0.000

we may have 216 (6*6*6) random combinations of coefficients for three objectives. Then, we can simulate these 216 combinations through ISGP II procedures.

Before applying ISGP II to these fuzzy MODM problem, we use the objectives in equation 4.4 (Max $f_1(\underline{x})$, Min $f_2(\underline{x})$ and Min $f_3(\underline{x})$ where $T_i = 1$) as constraints then go through the procedures to obtain a set of MAG and MAR, by varying the values of T_i we can get a set of MAG and MAR for each random combination. The problems can be formulated as follows:

$$(4.6) \quad \text{Min } a^j = \{ d_1^- + d_2^- + d_3^- \}$$

s.t.

$$\underline{x} \in X$$

$$f_1(\underline{x}) + .736 d_1^- - .736 d_1^+ = b_1$$

$$f_2(\underline{x}) - .657 d_2^- + .657 d_2^+ = b_2$$

$$f_3(\underline{x}) - 9.770 d_3^- + 9.770 d_3^+ = b_3$$

where $\underline{f}^* = (.781, .372, 2.898)$ and $\underline{f}^- = (.045, 1.029, 12.668)$.

In Table 4-4, solutions for decision variables, MAGs and MARs are presented at $T_1 = 1$, $T_2 = 1$ and $T_3 = \{ 0, .2, .4, .6, .8, 1 \}$. In Appendix B, solutions of all combinations are listed from $T_1 = 0$, $T_2 = 0$ to $T_1 = 1$, $T_2 = 1$ and $T_3 = \{ 0, .2, .4, .6, .8, 1 \}$ systematically.

Table 4-4(11) Solutions of Decision Variables
& MAGs & MARs at T1 = 1 ,T2 = 1
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	1.391	1.744	2.109	2.486	2.875	3.279
x5	2.727	2.296	1.852	1.392	0.917	0.425
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.067	0.145	0.224	0.307	0.392	0.481
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	18.174	17.835	17.485	17.123	16.749	16.361
x15	13.650	13.698	13.748	13.800	13.853	13.909
x16	6.765	6.813	6.863	6.915	6.968	7.024

MAG

f1	0.604	0.606	0.609	0.611	0.613	0.616
f2	0.530	0.528	0.526	0.524	0.521	0.519
f3	5.243	5.215	5.185	5.155	5.122	5.091
F1	4.958	4.960	4.962	4.964	4.967	4.969
F2	1.338	1.336	1.334	1.332	1.330	1.328
F3	55.453	54.252	53.049	51.846	50.640	49.436

MAR

f1	76.0%	76.3%	76.6%	76.9%	77.2%	77.6%
f2	76.0%	76.3%	76.6%	76.9%	77.2%	77.6%
f3	76.0%	76.3%	76.6%	76.9%	77.2%	77.6%

	f*	f-
f1	0.781	0.045
f2	0.372	1.029
f3	2.898	12.668

4.4.3 Decision Making

From the simulation, it is clear that the optimal decision exists with equilibrium of MARs which are highest in three objectives. Here we may conclude that when $T_i = 1$, $i = 1, 2, 3$, the solutions are optimal. In other words, once the decision makers decide the α -level for the possibilities of the uncertain situations, the optimal set of solutions exists at the solution with ultra-optimistic coefficients for all objectives. The optimal set of solutions for our bank model are as follows:

$$\underline{x} = (6.4, 1.04, 2.4, 3.279, .425, 0, 0, 4.675, 0, \\ .481, 2.6, 3.9, 0, 16.361, 13.909, 7.024)$$

$$\underline{f} = (.616, .519, 5.901)$$

$$\underline{E} = (4.969, 1.328, 49.436)$$

$$\text{MAR} = 77.6\% \text{ for all three objectives.}$$

If the decision makers do not satisfy with this set of solutions, they may go through the trade-off procedures in ISGP II as those presented in deterministic model in Chapter 3. They may want to choose a set of solutions which is possible and acceptable by them from some of these combinations. This is a problem of multiple attribute decision making (MADM). Hwang and Yoon [22] survey all the possible approaches for MADM problems.

We may select a set of solutions according to a given

set of T_i or F_i which is most likely for DMs. We may consider MARs as criteria since the equilibrium of MARs among objectives is a important element of ISGP II. We may also combine both criteria, a given T_i or F_i and a equilibrium of MARs, to select a possible and acceptable bank balance-sheet position.

CHAPTER 5

CONCLUSIONS

Mathematical programming has been incorporated with bank asset and liability management in order to obtain an optimal structure of balance sheet position over a multiperiod planning horizon.

In the deterministic models as specified in Chapter 3, which interest rates of portfolios are known and deposit flows are fixed, we suggest using an approach, Interactive Sequential Goal Programming II (ISGP II), to obtain a set of nondominated solutions with maximum achievable rate (MAR) for three objective. Through the trade-off procedure of ISGP II, we can also find the solutions which are possible to reach under limited resources and acceptable by the decision makers. Comparing our results with those obtained by Tayi and Leonard [37], one of their results is dominated by our result, Maximum Achievable Goal (MAG) with 67% Maximum Achievable Goal (MAR). This is clearly an advantage of applying ISGP II to bank model.

Since the real-world situations usually cannot be described precisely, the existing mathematical programming approaches cannot handle the problems well under uncertain situations. In our study, we propose a model which

incorporates the fuzzy set theory to handle the inherent uncertainty of banking problems and human aspects.

In Chapter four, we found that when the α value of α -level set is given by decision makers, the optimal structure of our fuzzy bank model will be the one with ultra-optimistic coefficients in all three objective.

In our model, we only focus on fuzzifying the coefficients of three objectives. The real systems are more complex in nature, they may be uncertain not just in objectives, but in constraints and resources. For example, the interest rate may vary due to the change of economic situations; the cash flows may vary by human behaviors; the regulations may also change unexpectedly. The further development of incorporating fuzzy set theory and fuzzy linear programming would be useful for handling and for optimizing these problems in general.

Another important aspect that has not been examined in this study is the problem of forecasting. How to decide the fuzzy intervals for the coefficients, and how to decide the α value of the possibilities of uncertain situations will be another problems for the decision makers.

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APPENDIX A

In the example of Chapter 3, the constraints (3.6) to (3.8) and auxiliary variables x_{14} , x_{15} and x_{16} reflect the specific nature of the capital-adequacy objective. These capital-adequacy constraints are based on the Analyzing Bank Capital (ABC) system used by the United States Federal Reserve Board to examine commercial banks during the 1960s. The formula for calculating the amount of capital that examiners are presumed to consider adequate takes into account not only the bank's assets, but the bank's liabilities as well.

The ABC system divides a commercial bank's portfolio into six categories with the decision a function of decreasing liquidity and increasing risk. The assets are defined as:

Cash assets	C
Very short-term assets	A_1
Governments 1-5 or equivalent	A_2
Minimum-risk assets	A_3
Intermediate assets	A_4
Portfolio assets	A_5

The first segment of capital requirement Q_1 is a function of the various assets plus a fixed amount, F :

$$(1) \quad Q_1 = .005 A_1 + .04 A_2 + .04 A_3 + .06 A_4 + .1 A_5 + F$$

Q_1 may be called a liquidity index since it decreased as the portfolio is shifted into more liquid assets. The second element is the liquidity needed, D_L , which is a function of the liabilities of the bank:

$$(2) \quad D_L = .47 (\text{demand deposits}) + .36 (\text{time deposits}) \\ (\text{deposits of banks and government}) + \\ (\text{borrowings})$$

If the amount of liquidity required, D_L , is more than the holdings of primary and secondary assets:

$$(3) \quad D_L > C + .995 A_1 + .96 A_2$$

then the additional capital, Q_2 , is required. The amount of additional capital required is a function of D_L and of the asset groups C through A_5 . Let:

$$(4) \quad M_1 = C + .995 A_1 + .96 A_2$$

$$(5) \quad M_2 = M_1 + .90 A_3$$

$$(6) \quad M_3 = M_2 + .85 A_4$$

The F.R.B. capital analysis defines Q_2 as follows:

If $D_L \leq M_1$

$$(7) \quad Q_2 = 0$$

If $M_1 < D_L \leq M_2$

$$(8) \quad Q_2 = .065 (D_L - M_1)$$

If $M_2 < D_L \leq M_3$

$$(9) \quad Q_2 = .065 (D_L - M_1) + .04 (D_L - M_2)$$

If $M_3 < D_L$

$$(10) \quad Q_2 = .065 (D_L - M_1) + .04 (D_L - M_2) \\ + .095 (D_L - M_3)$$

We can see that the required capital increases as D_L increases in relation to the other variables, M_1 , M_2 and M_3 . The total capital requirement, Q , is:

$$(11) \quad Q = Q_1 + Q_2$$

For our example, we can define a weighted average of a bank's assets, A , and a weighted average of a bank's liabilities, D_L , as:

$$(12) \quad A = .005 (x_2 + x_3) + .04 (x_4 + x_5) + .06 x_6 \\ + .01 (x_7 + x_8 + x_9 + x_{10})$$

$$(13) \quad D_L = .47 (\text{demand deposits}) + .36 (\text{savings deposits}) \\ + x_{11} + x_{12} + x_{13}$$

Define a function $E(D_L)$ which is an increasing function of D_L :

$$(14) \quad E(D_L) = V_R + V_M + V_I \quad \text{where } V_R, V_M, V_I \geq 0$$

$$(15) \quad V_R = .065 (D_L - x_1 - .995 x_2 - .995 x_3 - .96 x_4)$$

$$(16) \quad V_M = .040 (D_L - x_1 - .995 x_2 - .995 x_3 - .96 x_4 \\ - .90 x_5)$$

$$(17) \quad V_I = .095 (D_L - x_1 - .995 x_2 - .995 x_3 - .96 x_4 \\ - .90 x_5 - .85 x_6)$$

Equations (15) to (17) indicate that $E(D_L)$ increases when the deposit index D_L passes a critical point relative to the bank's primary and secondary asset reserves ($x_1 - x_6$). Once the critical point is passed, successive increases in the deposit index D_L force an increase in the ratio of capital to assets at an increasing rate.

Define the following auxiliary variables:

$$(18) \quad x_{14} = V_R / .065$$

$$(19) \quad x_{15} = V_M / .040$$

$$(20) \quad x_{16} = V_I / .095$$

From equations (18) to (20), we can obtain the equations (3.6), (3.7) and (3.8) by substituting \$40 million for demand deposits and \$20 million for savings deposits.

APPENDIX B

In this Appendix, we systematically present the solutions of decision variables at different T_i values where $T_i = \{ 0, .2, .4, .6, .8, 1 \}$ and $i = 1, 2$ and 3 for three objectives of bank model. In the tables, we also show the MAGs and MARs for each objective.

Table 4-4(00) Solutions of Decision Variables
& MAGs & MARs at T1 = 0 ,T2 = 0
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.770	3.770	3.770	3.770	3.770	3.770
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.511	2.511	2.511	2.511	2.511	2.511
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	2.882	2.882	2.882	2.882	2.882	2.882
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.926	16.926	16.926	16.926	16.926	16.926
x15	14.856	14.856	14.856	14.856	14.856	14.856
x16	7.971	7.971	7.971	7.971	7.971	7.971
MAG						
f1	0.436	0.436	0.436	0.436	0.436	0.436
f2	0.773	0.773	0.773	0.773	0.773	0.773
f3	7.914	7.725	7.537	7.348	7.160	6.971
F1	3.997	3.997	3.997	3.997	3.997	3.997
F2	1.928	1.928	1.928	1.928	1.928	1.928
F3	58.124	56.762	55.401	54.039	52.678	51.316
MAR						
f1	53.1%	53.1%	53.1%	53.1%	53.1%	53.1%
f2	39.0%	39.0%	39.0%	39.0%	39.0%	39.0%
f3	48.7%	50.6%	52.5%	54.5%	56.4%	58.3%
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(02) Solutions of Decision Variables
& MAGs & MARs at T1 = 0 ,T2 = .2
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.895	3.895	3.895	3.895	3.895	3.895
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.884	2.884	2.884	2.884	2.884	2.884
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.379	3.379	3.379	3.379	3.379	3.379
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	17.423	17.423	17.423	17.423	17.423	17.423
x15	15.353	15.353	15.353	15.353	15.353	15.353
x16	8.468	8.468	8.468	8.468	8.468	8.468
MAG						
f1	0.449	0.449	0.449	0.449	0.449	0.449
f2	0.761	0.761	0.761	0.761	0.761	0.761
f3	8.611	8.407	8.204	8.000	7.797	7.594
F1	4.010	4.010	4.010	4.010	4.010	4.010
F2	1.847	1.847	1.847	1.847	1.847	1.847
F3	58.821	57.444	56.068	54.691	53.315	51.939
MAR						
f1	54.8%	54.8%	54.8%	54.8%	54.8%	54.8%
f2	40.7%	40.7%	40.7%	40.7%	40.7%	40.7%
f3	41.5%	43.6%	45.7%	47.8%	49.9%	51.9%
f*						
f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(04) Solutions of Decision Variables
& MAGs & MARs at T1 = 0 ,T2 = .4
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.401	2.401
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.906	3.936	3.967	3.999	4.033	4.033
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.917	3.007	3.101	3.197	3.297	3.297
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.423	3.543	3.667	3.796	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	17.467	17.587	17.711	17.840	17.974	17.974
x15	15.397	15.517	15.641	15.770	15.904	15.904
x16	8.512	8.632	8.756	8.885	9.186	9.186
MAG						
f1	0.450	0.453	0.456	0.459	0.464	0.464
f2	0.716	0.724	0.732	0.740	0.753	0.753
f3	8.672	8.632	8.591	8.547	8.502	8.282
F1	4.011	4.014	4.017	4.020	4.026	4.026
F2	1.732	1.740	1.748	1.757	1.769	1.769
F3	58.882	57.669	56.455	55.238	54.020	52.627
MAR						
f1	55.0%	55.4%	55.8%	56.3%	57.0%	57.0%
f2	47.7%	46.5%	45.2%	43.9%	42.1%	42.1%
f3	40.9%	41.3%	41.7%	42.2%	42.6%	44.9%
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(06) Solutions of Decision Variables
& MAGs & MARs at T1 = 0 ,T2 = .6
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.401	2.408
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.906	3.936	3.967	3.999	4.034	4.075
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.917	3.007	3.101	3.197	3.299	3.041
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.423	3.543	3.667	3.796	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.036	0.203
x14	17.467	17.587	17.711	17.840	17.977	18.131
x15	15.397	15.517	15.641	15.770	15.907	16.061
x16	8.512	8.632	8.756	8.885	9.022	9.176
MAG						
f1	0.450	0.453	0.456	0.459	0.462	0.431
f2	0.667	0.674	0.682	0.690	0.698	0.700
f3	8.672	8.632	8.591	8.547	8.506	7.977
F1	4.011	4.014	4.017	4.020	4.024	3.993
F2	1.614	1.622	1.629	1.637	1.646	1.647
F3	58.882	57.669	56.455	55.238	54.024	52.322
MAR						
f1	55.0%	55.4%	55.8%	56.3%	56.7%	52.5%
f2	55.1%	54.0%	52.8%	51.6%	50.3%	50.0%
f3	40.9%	41.3%	41.7%	42.2%	42.6%	48.0%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(08) Solutions of Decision Variables
& MAGs & MARs at T1 = 0 ,T2 = .8
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.401	2.408
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.906	3.936	3.967	3.999	4.034	4.075
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.917	3.007	3.101	3.197	3.299	3.041
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.423	3.543	3.667	3.796	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.036	0.203
x14	17.467	17.587	17.711	17.840	17.977	18.131
x15	15.397	15.517	15.641	15.770	15.907	16.061
x16	8.512	8.632	8.756	8.885	9.022	9.176
MAG						
f1	0.450	0.453	0.456	0.459	0.462	0.431
f2	0.618	0.625	0.632	0.640	0.647	0.649
f3	8.672	8.632	8.591	8.547	8.506	7.977
F1	4.011	4.014	4.017	4.020	4.024	3.993
F2	1.496	1.503	1.510	1.517	1.525	1.527
F3	58.882	57.669	56.455	55.238	54.024	52.322
MAR						
f1	55.0%	55.4%	55.8%	56.3%	56.7%	52.5%
f2	62.5%	61.5%	60.4%	59.3%	58.1%	57.8%
f3	40.9%	41.3%	41.7%	42.2%	42.6%	48.0%
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(01) Solutions of Decision Variables
& MAGs & MARs at T1 = 0 ,T2 = 1
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.401	2.408
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.906	3.936	3.967	3.999	4.034	4.075
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.917	3.007	3.101	3.197	3.299	3.041
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.423	3.543	3.667	3.796	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.036	0.203
x14	17.467	17.587	17.711	17.840	17.977	18.131
x15	15.397	15.517	15.641	15.770	15.907	16.061
x16	8.512	8.632	8.756	8.885	9.022	9.176
MAG						
f1	0.450	0.453	0.456	0.459	0.462	0.431
f2	0.570	0.576	0.583	0.589	0.596	0.598
f3	8.672	8.632	8.591	8.547	8.506	7.977
F1	4.011	4.014	4.017	4.020	4.024	3.993
F2	1.378	1.385	1.391	1.398	1.405	1.407
F3	58.882	57.669	56.455	55.238	54.024	52.322
MAR						
f1	55.0%	55.4%	55.8%	56.3%	56.7%	52.5%
f2	69.9%	68.9%	68.0%	66.9%	65.8%	65.6%
f3	40.9%	41.3%	41.7%	42.2%	42.6%	48.0%
f*						
f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(20) Solutions of Decision Variables
& MAGs & MARs at T1 = .2, T2 = 0
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.470	3.470	3.470	3.470	3.470	3.470
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.611	1.611	1.611	1.611	1.611	1.611
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	1.681	1.681	1.681	1.681	1.681	1.681
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.725	15.725	15.725	15.725	15.725	15.725
x15	13.655	13.655	13.655	13.655	13.655	13.655
x16	6.770	6.770	6.770	6.770	6.770	6.770
MAG						
f1	0.433	0.433	0.433	0.433	0.433	0.433
f2	0.682	0.682	0.682	0.682	0.682	0.682
f3	6.234	6.081	5.929	5.776	5.624	5.471
F1						
F2	1.837	1.837	1.837	1.837	1.837	1.837
F3	56.444	55.118	53.793	52.467	51.142	49.816
MAR						
f1	52.7%	52.7%	52.7%	52.7%	52.7%	52.7%
f2	52.7%	52.7%	52.7%	52.7%	52.7%	52.7%
f3	65.9%	67.4%	69.0%	70.5%	72.1%	73.7%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(22) Solutions of Decision Variables
& MAGs & MARs at T1 = .2, T2 = .2
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.574	3.574	3.574	3.574	3.574	3.574
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.922	1.922	1.922	1.922	1.922	1.922
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	2.096	2.096	2.096	2.096	2.096	2.096
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.140	16.140	16.140	16.140	16.140	16.140
x15	14.070	14.070	14.070	14.070	14.070	14.070
x16	7.185	7.185	7.185	7.185	7.185	7.185
MAG						
f1	0.446	0.446	0.446	0.446	0.446	0.446
f2	0.671	0.671	0.671	0.671	0.671	0.671
f3	6.814	6.650	6.485	6.320	6.155	5.990
F1	4.166	4.166	4.166	4.166	4.166	4.166
F2	1.757	1.757	1.757	1.757	1.757	1.757
F3	57.024	55.687	54.349	53.011	51.673	50.335
MAR						
f1	54.5%	54.5%	54.5%	54.5%	54.5%	54.5%
f2	54.5%	54.5%	54.5%	54.5%	54.5%	54.5%
f3	59.9%	61.6%	63.3%	65.0%	66.7%	68.4%
f*						
f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(24) Solutions of Decision Variables
& MAGs & MARs at T1 = .2, T2 = .4
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.647	3.671	3.688	3.688	3.688	3.688
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.140	2.213	2.263	2.263	2.263	2.263
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	2.387	2.485	2.551	2.551	2.551	2.551
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.631	16.529	16.595	16.595	16.595	16.595
x15	14.361	14.459	14.525	14.525	14.525	14.525
x16	7.476	7.574	7.640	7.640	7.640	7.640
MAG						
f1	0.455	0.458	0.460	0.460	0.460	0.460
f2	0.650	0.654	0.658	0.658	0.658	0.658
f3	7.222	7.181	7.094	6.916	6.737	6.559
F1	4.175	4.178	4.180	4.180	4.180	4.180
F2	1.667	1.670	1.675	1.675	1.675	1.675
F3	57.432	56.218	54.958	53.607	52.255	50.904
MAR						
f1	55.7%	56.1%	56.4%	56.4%	56.4%	56.4%
f2	57.7%	57.1%	56.4%	56.4%	56.4%	56.4%
f3	55.7%	56.2%	57.0%	58.9%	60.7%	62.5%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(26) Solutions of Decision Variables
& MAGs & MARs at T1 = .2, T2 = .6
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.647	3.671	3.696	3.722	3.749	3.777
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.140	2.213	2.289	2.367	2.448	2.532
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	2.387	2.485	2.585	2.689	2.797	2.909
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.631	16.529	16.629	16.733	16.841	16.953
x15	14.361	14.459	14.559	14.663	14.771	14.883
x16	7.476	7.574	7.674	7.778	7.886	7.998
MAG						
f1	0.455	0.458	0.461	0.465	0.468	0.472
f2	0.606	0.609	0.616	0.622	0.629	0.635
f3	7.222	7.181	7.140	7.097	7.052	7.006
F1	4.175	4.178	4.181	4.184	4.188	4.191
F2	1.553	1.556	1.563	1.569	1.576	1.583
F3	57.432	56.218	55.004	53.788	52.570	51.351
MAR						
f1	55.7%	56.1%	56.6%	57.0%	57.5%	58.0%
f2	64.4%	63.9%	62.9%	62.0%	60.9%	59.9%
f3	55.7%	56.2%	56.6%	57.0%	57.5%	57.9%
f*						
f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(28) Solutions of Decision Variables
& MAGs & MARs at T1 = .2, T2 = .8
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.647	3.671	3.696	3.722	3.749	3.777
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.140	2.213	2.289	2.367	2.448	2.532
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	2.387	2.485	2.585	2.689	2.797	2.909
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.631	16.529	16.629	16.733	16.841	16.953
x15	14.361	14.459	14.559	14.663	14.771	14.883
x16	7.476	7.574	7.674	7.778	7.886	7.998
MAG						
f1	0.455	0.458	0.461	0.465	0.468	0.472
f2	0.562	0.565	0.571	0.577	0.583	0.589
f3	7.222	7.181	7.140	7.097	7.052	7.006
F1	4.175	4.178	4.181	4.184	4.188	4.191
F2	1.440	1.443	1.448	1.454	1.460	1.467
F3	57.432	56.218	55.004	53.788	52.570	51.351
MAR						
f1	55.7%	56.1%	56.6%	57.0%	57.5%	58.0%
f2	71.1%	70.6%	69.8%	68.9%	67.9%	67.0%
f3	55.7%	56.2%	56.6%	57.0%	57.5%	57.9%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(21) Solutions of Decision Variables
& MAGs & MARs at T1 = .2, T2 = 1
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.647	3.671	3.696	3.722	3.749	3.777
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	2.140	2.213	2.289	2.367	2.448	2.532
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	2.387	2.485	2.585	2.689	2.797	2.909
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.631	16.529	16.629	16.733	16.841	16.953
x15	14.361	14.459	14.559	14.663	14.771	14.883
x16	7.476	7.574	7.674	7.778	7.886	7.998
.						
MAG						
f1	0.455	0.458	0.461	0.465	0.468	0.472
f2	0.518	0.520	0.526	0.531	0.537	0.543
f3	7.222	7.181	7.140	7.097	7.052	7.006
F1	4.175	4.178	4.181	4.184	4.188	4.191
F2	1.326	1.329	1.334	1.340	1.345	1.351
F3	57.432	56.218	55.004	53.788	52.570	51.351
MAR						
f1	55.7%	56.1%	56.6%	57.0%	57.5%	58.0%
f2	77.8%	77.4%	76.6%	75.8%	74.9%	74.0%
f3	55.7%	56.2%	56.6%	57.0%	57.5%	57.9%
.						
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(40) Solutions of Decision Variables
& MAGs & MARs at T1 = .4, T2 = 0
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.414	3.414	3.414	3.414	3.414	3.414
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.441	1.441	1.441	1.441	1.441	1.441
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	1.455	1.455	1.455	1.455	1.455	1.455
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.499	15.499	15.499	15.499	15.499	15.499
x15	13.429	13.429	13.429	13.429	13.429	13.429
x16	6.544	6.544	6.544	6.544	6.544	6.544
MAG						
f1	0.452	0.452	0.452	0.452	0.452	0.452
f2	0.666	0.666	0.666	0.666	0.666	0.666
f3	5.917	5.771	5.626	5.480	5.334	5.189
F1	4.330	4.330	4.330	4.330	4.330	4.330
F2	1.821	1.821	1.821	1.821	1.821	1.821
F3	56.127	54.808	53.490	52.171	50.852	49.534
MAR						
f1	55.3%	55.3%	55.3%	55.3%	55.3%	55.3%
f2	55.3%	55.3%	55.3%	55.3%	55.3%	55.3%
f3	69.1%	70.6%	72.1%	73.6%	75.1%	76.6%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(42) Solutions of Decision Variables
& MAGs & MARs at T1 = .4, T2 = .2
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.510	3.510	3.510	3.510	3.510	3.510
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.729	1.729	1.729	1.729	1.729	1.729
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	1.838	1.838	1.838	1.838	1.838	1.838
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.882	15.882	15.882	15.882	15.882	15.882
x15	13.812	13.812	13.812	13.812	13.812	13.812
x16	6.927	6.927	6.927	6.927	6.927	6.927
MAG						
f1	0.467	0.467	0.467	0.467	0.467	0.467
f2	0.653	0.653	0.653	0.653	0.653	0.653
f3	6.455	6.297	6.140	5.983	5.826	5.669
F1	4.345	4.345	4.345	4.345	4.345	4.345
F2	1.738	1.738	1.738	1.738	1.738	1.738
F3	56.665	55.334	54.004	52.674	51.344	50.014
MAR						
f1	57.3%	57.3%	57.3%	57.3%	57.3%	57.3%
f2	57.3%	57.3%	57.3%	57.3%	57.3%	57.3%
f3	63.6%	65.2%	66.8%	68.4%	70.0%	71.6%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(44) Solutions of Decision Variables
& MAGs & MARs at T1 = .4, T2 = .4
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.292	0.008	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.668	3.616	3.614	3.614	3.614	3.614
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.912	2.039	2.043	2.043	2.043	2.043
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	2.472	2.263	2.257	2.257	2.257	2.257
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.516	16.307	16.301	16.301	16.301	16.301
x15	14.183	14.230	14.231	14.231	14.231	14.231
x16	7.298	7.345	7.346	7.346	7.346	7.346
MAG						
f1	0.479	0.482	0.482	0.482	0.482	0.482
f2	0.641	0.639	0.639	0.639	0.639	0.639
f3	6.903	6.866	6.700	6.531	6.361	6.191
F1	4.358	4.360	4.360	4.360	4.360	4.360
F2	1.658	1.655	1.655	1.655	1.655	1.655
F3	57.113	55.903	54.564	53.222	51.879	50.536
MAR						
f1	59.0%	59.4%	59.4%	59.4%	59.4%	59.4%
f2	59.0%	59.4%	59.4%	59.4%	59.4%	59.4%
f3	59.0%	59.4%	61.1%	62.8%	64.6%	66.3%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(46) Solutions of Decision Variables
& MAGs & MARs at T1 = .4, T2 = .6
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	1.362	1.485	1.184	0.875	0.557	0.229
x6	0.291	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.025	4.004	3.948	3.891	3.832	3.771
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.622	1.727	1.861	1.998	2.140	2.285
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.900	3.817	3.594	3.364	3.128	2.886
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	17.944	17.861	17.638	17.408	17.172	16.930
x15	14.649	14.454	14.502	14.551	14.601	14.653
x16	7.516	7.569	7.617	7.666	7.716	7.768
MAG						
f1	0.482	0.485	0.488	0.491	0.494	0.497
f2	0.638	0.636	0.633	0.631	0.628	0.625
f3	6.861	6.823	6.786	6.747	6.709	6.667
F1	4.361	4.363	4.366	4.369	4.372	4.375
F2	1.586	1.583	1.581	1.578	1.575	1.573
F3	57.071	55.860	54.650	53.438	52.227	51.012
MAR						
f1	59.4%	59.8%	60.2%	60.6%	61.0%	61.4%
f2	59.4%	59.8%	60.2%	60.6%	61.0%	61.4%
f3	59.4%	59.8%	60.2%	60.6%	61.0%	61.4%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(48) Solutions of Decision Variables
& MAGs & MARs at T1 = .4, T2 = .8
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	1.662	1.575	1.485	1.393	1.298	1.199
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.025	4.025	4.025	4.025	4.025	4.025
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.613	1.700	1.790	1.882	1.977	2.076
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	17.944	17.944	17.944	17.944	17.944	17.944
x15	15.874	15.874	15.874	15.874	15.874	15.874
x16	7.576	7.650	7.726	7.805	7.886	7.970
MAG						
f1	0.483	0.487	0.490	0.493	0.497	0.500
f2	0.608	0.609	0.612	0.614	0.616	0.618
f3	6.847	6.806	6.764	6.719	6.673	6.626
F1	4.362	4.365	4.368	4.371	4.375	4.378
F2	1.485	1.487	1.489	1.491	1.494	1.496
F3	57.057	55.843	54.628	53.410	52.191	50.971
MAR						
f1	59.6%	60.0%	60.4%	60.9%	61.4%	61.8%
f2	64.2%	63.9%	63.5%	63.2%	62.9%	62.5%
f3	59.6%	60.0%	60.4%	60.9%	61.4%	61.8%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(41) Solutions of Decision Variables
& MAGs & MARs at T1 = .4, T2 = 1
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	1.662	1.575	1.485	1.393	1.298	1.199
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.025	4.025	4.025	4.025	4.025	4.025
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.613	1.700	1.790	1.882	1.977	2.076
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	17.944	17.944	17.944	17.944	17.944	17.944
x15	15.874	15.874	15.874	15.874	15.874	15.874
x16	7.576	7.650	7.726	7.805	7.886	7.970
MAG						
f1	0.483	0.487	0.490	0.493	0.497	0.500
f2	0.560	0.562	0.564	0.565	0.567	0.570
f3	6.847	6.806	6.764	6.719	6.673	6.626
F1	4.362	4.365	4.368	4.371	4.375	4.378
F2	1.368	1.370	1.372	1.374	1.376	1.378
F3	57.057	55.843	54.628	53.410	52.191	50.971
MAR						
f1	59.6%	60.0%	60.4%	60.9%	61.4%	61.8%
f2	71.4%	71.1%	70.9%	70.6%	70.3%	69.9%
f3	59.6%	60.0%	60.4%	60.9%	61.4%	61.8%
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(60) Solutions of Decision Variables
& MAGs & MARs at $T_1 = .6, T_2 = 0$
& $T_3 = \{ 0, .2, .4, .6, .8, 1 \}$

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.362	3.362	3.362	3.362	3.362	3.362
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.287	1.287	1.287	1.287	1.287	1.287
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	1.249	1.249	1.249	1.249	1.249	1.249
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.293	15.293	15.293	15.293	15.293	15.293
x15	13.223	13.223	13.223	13.223	13.223	13.223
x16	6.338	6.338	6.338	6.338	6.338	6.338
MAG						
f1	0.469	0.469	0.469	0.469	0.469	0.469
f2	0.650	0.650	0.650	0.650	0.650	0.650
f3	5.629	5.489	5.350	5.210	5.071	4.931
F1	4.506	4.506	4.506	4.506	4.506	4.506
F2	1.805	1.805	1.805	1.805	1.805	1.805
F3	55.839	54.526	53.214	51.901	50.589	49.276
MAR						
f1	57.7%	57.7%	57.7%	57.7%	57.7%	57.7%
f2	57.7%	57.7%	57.7%	57.7%	57.7%	57.7%
f3	72.1%	73.5%	74.9%	76.3%	77.8%	79.2%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(62) Solutions of Decision Variables
& MAGs & MARs at T1 = .6, T2 = .2
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	4.760	4.760	4.760	4.760	4.760	4.760
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.025	4.025	4.025	4.025	4.025	4.025
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.915	0.915	0.915	0.915	0.915	0.915
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.596	15.596	15.596	15.596	15.596	15.596
x15	13.526	13.526	13.526	13.526	13.526	13.526
x16	6.641	6.641	6.641	6.641	6.641	6.641
MAG						
f1	0.485	0.485	0.485	0.485	0.485	0.485
f2	0.636	0.636	0.636	0.636	0.636	0.636
f3	5.800	5.652	5.504	5.355	5.207	5.059
F1	4.522	4.522	4.522	4.522	4.522	4.522
F2	1.722	1.722	1.722	1.722	1.722	1.722
F3	56.010	54.689	53.368	52.046	50.725	49.404
MAR						
f1	59.8%	59.8%	59.8%	59.8%	59.8%	59.8%
f2	59.8%	59.8%	59.8%	59.8%	59.8%	59.8%
f3	70.3%	71.8%	73.3%	74.8%	76.4%	77.9%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(64) Solutions of Decision Variables
& MAGs & MARs at T1 = .6, T2 = .4
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	4.358	4.358	4.358	4.358	4.358	4.358
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.025	4.025	4.025	4.025	4.025	4.025
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.317	1.317	1.317	1.317	1.317	1.317
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.996	15.996	15.996	15.996	15.996	15.996
x15	13.926	13.926	13.926	13.926	13.926	13.926
x16	7.041	7.041	7.041	7.041	7.041	7.041
MAG						
f1	0.502	0.502	0.502	0.502	0.502	0.502
f2	0.621	0.621	0.621	0.621	0.621	0.621
f3	6.403	6.243	6.082	5.922	5.762	5.602
F1	4.538	4.538	4.538	4.538	4.538	4.538
F2	1.638	1.638	1.638	1.638	1.638	1.638
F3	56.613	55.280	53.946	52.613	51.280	49.947
MAR						
f1	62.1%	62.1%	62.1%	62.1%	62.1%	62.1%
f2	62.1%	62.1%	62.1%	62.1%	62.1%	62.1%
f3	64.1%	65.8%	67.4%	69.0%	70.7%	72.3%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(66) Solutions of Decision Variables
& MAGs & MARs at T1 = .6, T2 = .6
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.565	0.305	0.040	0.000	0.000	0.000
x3	2.400	2.400	2.400	3.077	3.901	3.922
x4	2.681	2.239	1.788	0.954	0.023	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.378	4.216	4.050	4.025	4.025	4.025
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	1.088	1.303	1.523	1.644	1.751	1.753
x11	1.413	0.763	0.101	0.000	0.000	0.000
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.221	16.254	16.288	16.354	16.428	16.430
x15	14.151	14.184	14.218	14.284	14.358	14.360
x16	7.266	7.299	7.333	7.399	7.473	7.475
MAG						
f1	0.513	0.515	0.517	0.518	0.520	0.520
f2	0.611	0.609	0.608	0.606	0.605	0.605
f3	6.448	6.427	6.405	6.383	6.361	6.190
F1	4.550	4.552	4.553	4.555	4.557	4.557
F2	1.558	1.556	1.555	1.553	1.552	1.552
F3	56.658	55.464	54.269	53.074	51.879	50.535
MAR						
f1	63.6%	63.9%	64.1%	64.3%	64.6%	64.6%
f2	63.7%	63.9%	64.1%	64.3%	64.6%	64.6%
f3	63.7%	63.9%	64.1%	64.3%	64.6%	66.3%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(68) Solutions of Decision Variables
& MAGs & MARs at T1 = .6, T2 = .8
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	2.400	2.400	2.400	2.400	2.400	2.851
x4	1.065	1.461	1.870	2.293	2.730	2.436
x5	2.305	1.820	1.318	0.800	0.264	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.815	0.904	0.997	1.092	1.191	1.298
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	18.486	18.106	17.714	17.308	16.888	16.722
x15	14.342	14.399	14.457	14.518	14.580	14.652
x16	7.457	7.514	7.572	7.633	7.695	7.767
MAG						
f1	0.520	0.522	0.525	0.528	0.530	0.533
f2	0.605	0.603	0.601	0.598	0.596	0.594
f3	6.365	6.331	6.298	6.261	6.225	6.194
F1						
F1	4.556	4.559	4.562	4.564	4.567	4.569
F2	1.483	1.481	1.478	1.476	1.473	1.471
F3	56.575	55.368	54.162	52.952	51.743	50.539
MAR						
f1	64.5%	64.9%	65.2%	65.6%	66.0%	66.3%
f2	64.5%	64.9%	65.2%	65.6%	66.0%	66.3%
f3	64.5%	64.9%	65.2%	65.6%	65.9%	66.3%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(61) Solutions of Decision Variables
& MAGs & MARs at T1 = .6, T2 = 1
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.385	0.786	1.199	1.626	2.067	2.523
x6	3.019	2.532	2.029	1.510	0.973	0.418
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.781	0.867	0.957	1.049	1.145	1.244
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	19.509	19.509	19.509	19.509	19.509	19.509
x15	17.092	16.732	16.360	15.976	15.579	15.168
x16	7.641	7.695	7.750	7.808	7.867	7.928
MAG						
f1	0.524	0.526	0.529	0.532	0.535	0.538
f2	0.602	0.599	0.597	0.594	0.591	0.589
f3	6.314	6.277	6.240	6.201	6.162	6.121
F1	4.560	4.563	4.566	4.569	4.572	4.575
F2	1.410	1.408	1.405	1.403	1.400	1.397
F3	56.524	55.314	54.104	52.892	51.680	50.466
MAR						
f1	65.0%	65.4%	65.8%	66.2%	66.6%	67.0%
f2	65.0%	65.4%	65.8%	66.2%	66.6%	67.0%
f3	65.0%	65.4%	65.8%	66.2%	66.6%	67.0%
f*						
f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(80) Solutions of Decision Variables
& MAGs & MARs at T1 = .8, T2 = 0
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.275	0.275	0.275	0.275	0.275	0.275
x3	5.916	5.916	5.916	5.916	5.916	5.916
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.197	4.197	4.197	4.197	4.197	4.197
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.000	0.000	0.000	0.000	0.000	0.000
x11	0.688	0.688	0.688	0.688	0.688	0.688
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	14.860	14.860	14.860	14.860	14.860	14.860
x15	12.790	12.790	12.790	12.790	12.790	12.790
x16	5.905	5.905	5.905	5.905	5.905	5.905
MAG						
f1	0.498	0.498	0.498	0.498	0.498	0.498
f2	0.625	0.625	0.625	0.625	0.625	0.625
f3	4.617	4.491	4.365	4.239	4.113	3.987
F1	4.693	4.693	4.693	4.693	4.693	4.693
F2	1.780	1.780	1.780	1.780	1.780	1.780
F3	54.827	53.528	52.229	50.930	49.631	48.332
MAR						
f1	61.5%	61.5%	61.5%	61.5%	61.5%	61.5%
f2	61.5%	61.5%	61.5%	61.5%	61.5%	61.5%
f3	82.4%	83.7%	85.0%	86.3%	87.6%	88.9%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(82) Solutions of Decision Variables
& MAGs & MARs at T1 = .8, T2 = .2
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.740	0.740	0.740	0.740	0.740	0.740
x3	6.322	6.322	6.322	6.322	6.322	6.322
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.487	4.487	4.487	4.487	4.487	4.487
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.000	0.000	0.000	0.000	0.000	0.000
x11	1.849	1.849	1.849	1.849	1.849	1.849
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.155	15.155	15.155	15.155	15.155	15.155
x15	13.085	13.085	13.085	13.085	13.085	13.085
x16	6.200	6.200	6.200	6.200	6.200	6.200
MAG						
f1	0.516	0.516	0.516	0.516	0.516	0.516
f2	0.609	0.609	0.609	0.609	0.609	0.609
f3	4.936	4.801	4.666	4.532	4.397	4.263
F1	4.711	4.711	4.711	4.711	4.711	4.711
F2	1.694	1.694	1.694	1.694	1.694	1.694
F3	55.146	53.838	52.530	51.223	49.915	48.608
MAR						
f1	64.0%	64.0%	64.0%	64.0%	64.0%	64.0%
f2	64.0%	64.0%	64.0%	64.0%	64.0%	64.0%
f3	79.1%	80.5%	81.9%	83.3%	84.7%	86.0%
f*						
f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(84) Solutions of Decision Variables
& MAGs & MARs at T1 = .8, T2 = .4
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	6.428	6.428	6.428	6.428	6.428	6.428
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.157	0.157	0.157	0.157	0.157	0.157
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.502	15.502	15.502	15.502	15.502	15.502
x15	13.432	13.432	13.432	13.432	13.432	13.432
x16	6.547	6.547	6.547	6.547	6.547	6.547
MAG						
f1	0.534	0.534	0.534	0.534	0.534	0.534
f2	0.593	0.593	0.593	0.593	0.593	0.593
f3	5.378	5.233	5.088	4.943	4.798	4.653
F1	4.729	4.729	4.729	4.729	4.729	4.729
F2	1.609	1.609	1.609	1.609	1.609	1.609
F3	55.588	54.270	52.952	51.634	50.316	48.998
MAR						
f1	66.4%	66.4%	66.4%	66.4%	66.4%	66.4%
f2	66.4%	66.4%	66.4%	66.4%	66.4%	66.4%
f3	74.6%	76.1%	77.6%	79.1%	80.6%	82.0%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(86) Solutions of Decision Variables
& MAGs & MARs at T1 = .8, T2 = .6
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	5.865	6.005	6.005	6.005	6.005	6.005
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.539	0.580	0.580	0.580	0.580	0.580
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.894	15.923	15.923	15.923	15.923	15.923
x15	13.824	13.853	13.853	13.853	13.853	13.853
x16	6.939	6.968	6.968	6.968	6.968	6.968
MAG						
f1	0.539	0.552	0.552	0.552	0.552	0.552
f2	0.575	0.577	0.577	0.577	0.577	0.577
f3	5.951	5.855	5.697	5.540	5.382	5.224
F1	4.734	4.747	4.747	4.747	4.747	4.747
F2	1.522	1.524	1.524	1.524	1.524	1.524
F3	56.161	54.892	53.561	52.231	50.900	49.569
MAR						
f1	67.1%	68.8%	68.8%	68.8%	68.8%	68.8%
f2	69.2%	68.8%	68.8%	68.8%	68.8%	68.8%
f3	68.8%	69.7%	71.3%	73.0%	74.6%	76.2%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(88) Solutions of Decision Variables
& MAGs & MARs at T1 = .8, T2 = .8
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	2.400	2.400	2.414	3.165	3.944	4.752
x4	2.809	3.179	3.539	2.690	1.809	0.895
x5	0.914	0.460	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.462	0.546	0.632	0.730	0.832	0.938
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	16.813	16.457	16.098	16.166	16.236	16.310
x15	13.920	13.973	14.028	14.096	14.166	14.240
x16	7.035	7.088	7.143	7.211	7.281	7.355
MAG						
f1	0.560	0.562	0.564	0.566	0.568	0.569
f2	0.570	0.567	0.565	0.564	0.562	0.561
f3	5.836	5.805	5.772	5.751	5.730	5.708
F1	4.755	4.757	4.759	4.761	4.763	4.764
F2	1.447	1.445	1.443	1.442	1.440	1.439
F3	56.046	54.842	53.636	52.442	51.248	50.053
MAR						
f1	69.9%	70.3%	70.6%	70.8%	71.0%	71.2%
f2	69.9%	70.3%	70.6%	70.8%	71.0%	71.2%
f3	69.9%	70.2%	70.6%	70.8%	71.0%	71.2%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(81) Solutions of Decision Variables
& MAGs & MARs at T1 = .8, T2 = 1
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	0.000	0.000	0.210	0.613	1.029	1.460
x5	3.093	3.467	3.389	2.897	2.389	1.863
x6	0.671	0.216	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.420	0.501	0.586	0.675	0.766	0.861
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	19.509	19.509	19.308	18.921	18.521	18.107
x15	14.655	14.319	14.188	14.243	14.301	14.360
x16	7.199	7.250	7.303	7.358	7.416	7.475
MAG						
f1	0.564	0.567	0.569	0.572	0.574	0.577
f2	0.565	0.563	0.561	0.559	0.556	0.554
f3	5.773	5.739	5.706	5.674	5.639	5.604
F1	4.759	4.762	4.764	4.767	4.769	4.772
F2	1.374	1.372	1.369	1.367	1.365	1.362
F3	55.983	54.776	53.570	52.365	51.157	49.949
MAR						
f1	70.6%	70.9%	71.3%	71.6%	71.9%	72.3%
f2	70.6%	70.9%	71.3%	71.6%	71.9%	72.3%
f3	70.6%	70.9%	71.3%	71.6%	71.9%	72.3%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(10) Solutions of Decision Variables
& MAGs & MARs at T1 = 1 ,T2 = 0
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.000	0.000	0.000	0.000	0.000	0.000
x3	5.442	5.442	5.442	5.442	5.442	5.442
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	3.947	3.947	3.947	3.947	3.947	3.947
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.000	0.000	0.000	0.000	0.000	0.000
x11	0.000	0.000	0.000	0.000	0.000	0.000
x12	3.358	3.358	3.358	3.358	3.358	3.358
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	14.607	14.607	14.607	14.607	14.607	14.607
x15	12.537	12.537	12.537	12.537	12.537	12.537
x16	5.652	5.652	5.652	5.652	5.652	5.652
MAG						
f1	0.531	0.531	0.531	0.531	0.531	0.531
f2	0.604	0.604	0.604	0.604	0.604	0.604
f3	4.342	4.223	4.105	3.986	3.868	3.750
F1	4.885	4.885	4.885	4.885	4.885	4.885
F2	1.759	1.759	1.759	1.759	1.759	1.759
F3	54.552	53.260	51.969	50.677	49.386	48.095
MAR						
f1	66.1%	66.1%	66.1%	66.1%	66.1%	66.1%
f2	64.6%	64.6%	64.6%	64.6%	64.6%	64.6%
f3	85.2%	86.4%	87.6%	88.9%	90.1%	91.3%
f* f-						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(12) Solutions of Decision Variables
& MAGs & MARs at T1 = 1 ,T2 = .2
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.254	0.254	0.254	0.254	0.254	0.254
x3	5.897	5.897	5.897	5.897	5.897	5.897
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.182	4.182	4.182	4.182	4.182	4.182
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.000	0.000	0.000	0.000	0.000	0.000
x11	0.634	0.634	0.634	0.634	0.634	0.634
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	14.846	14.846	14.846	14.846	14.846	14.846
x15	12.776	12.776	12.776	12.776	12.776	12.776
x16	5.891	5.891	5.891	5.891	5.891	5.891
MAG						
f1	0.542	0.542	0.542	0.542	0.542	0.542
f2	0.586	0.586	0.586	0.586	0.586	0.586
f3	4.600	4.475	4.349	4.224	4.098	3.973
F1	4.895	4.895	4.895	4.895	4.895	4.895
F2	1.671	1.671	1.671	1.671	1.671	1.671
F3	54.810	53.512	52.213	50.915	49.616	48.318
MAR						
f1	67.5%	67.5%	67.5%	67.5%	67.5%	67.5%
f2	67.5%	67.5%	67.5%	67.5%	67.5%	67.5%
f3	82.6%	83.9%	85.1%	86.4%	87.7%	89.0%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(14) Solutions of Decision Variables
& MAGs & MARs at T1 = 1 ,T2 = .4
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	0.667	0.667	0.667	0.667	0.667	0.667
x3	6.259	6.259	6.259	6.259	6.259	6.259
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.442	4.442	4.442	4.442	4.442	4.442
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.000	0.000	0.000	0.000	0.000	0.000
x11	1.667	1.667	1.667	1.667	1.667	1.667
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.108	15.108	15.108	15.108	15.108	15.108
x15	13.038	13.038	13.038	13.038	13.038	13.038
x16	6.153	6.153	6.153	6.153	6.153	6.153
MAG						
f1	0.564	0.564	0.564	0.564	0.564	0.564
f2	0.566	0.566	0.566	0.566	0.566	0.566
f3	4.886	4.753	4.620	4.486	4.353	4.220
F1	4.917	4.917	4.917	4.917	4.917	4.917
F2	1.582	1.582	1.582	1.582	1.582	1.582
F3	55.096	53.790	52.484	51.177	49.871	48.565
MAR						
f1	70.5%	70.5%	70.5%	70.5%	70.5%	70.5%
f2	70.5%	70.5%	70.5%	70.5%	70.5%	70.5%
f3	79.6%	81.0%	82.4%	83.7%	85.1%	86.5%
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(16) Solutions of Decision Variables
& MAGs & MARs at T1 = 1 ,T2 = .6
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	6.520	6.520	6.520	6.520	6.520	6.520
x4	0.000	0.000	0.000	0.000	0.000	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.065	0.065	0.065	0.065	0.065	0.065
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.410	15.410	15.410	15.410	15.410	15.410
x15	13.430	13.430	13.430	13.430	13.430	13.430
x16	6.455	6.455	6.455	6.455	6.455	6.455
MAG						
f1	0.586	0.586	0.586	0.586	0.586	0.586
f2	0.546	0.546	0.546	0.546	0.546	0.546
f3	5.240	5.098	4.956	4.813	4.671	4.529
F1	4.940	4.940	4.940	4.940	4.940	4.940
F2	1.494	1.494	1.494	1.494	1.494	1.494
F3	55.450	54.135	52.820	51.504	50.189	48.874
MAR						
f1	73.6%	73.6%	73.6%	73.6%	73.6%	73.6%
f2	73.4%	73.4%	73.4%	73.4%	73.4%	73.4%
f3	76.0%	77.5%	78.9%	80.4%	81.9%	83.3%
f*						
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(18) Solutions of Decision Variables
& MAGs & MARs at T1 = 1 ,T2 = .8
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
	-----	-----	-----	-----	-----	-----
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	3.135	3.788	4.466	5.168	5.897	6.082
x4	3.333	2.594	1.828	1.033	0.209	0.000
x5	0.000	0.000	0.000	0.000	0.000	0.000
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.118	0.203	0.292	0.383	0.479	0.503
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	15.579	15.638	15.699	15.763	15.829	15.845
x15	13.509	13.568	13.629	13.693	13.759	13.775
x16	6.624	6.683	6.744	6.808	6.874	6.890
MAG						
f1	0.599	0.600	0.601	0.603	0.604	0.605
f2	0.535	0.534	0.532	0.531	0.530	0.529
f3	5.320	5.301	5.282	5.262	5.243	5.120
F1	4.952	4.953	4.955	4.956	4.958	4.958
F2	1.413	1.411	1.410	1.409	1.407	1.407
F3	55.530	54.338	53.146	51.953	50.761	49.465
MAR						
f1	75.2%	75.4%	75.6%	75.8%	76.0%	76.1%
f2	75.2%	75.4%	75.6%	75.8%	76.0%	76.1%
f3	75.2%	75.4%	75.6%	75.8%	76.0%	77.3%
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

Table 4-4(11) Solutions of Decision Variables
& MAGs & MARs at T1 = 1 ,T2 = 1
& T3 = { 0, .2, .4, .6, .8, 1 }

T3 =	0.000	0.200	0.400	0.600	0.800	1.000
x1	6.400	6.400	6.400	6.400	6.400	6.400
x2	1.040	1.040	1.040	1.040	1.040	1.040
x3	2.400	2.400	2.400	2.400	2.400	2.400
x4	1.391	1.744	2.109	2.486	2.875	3.279
x5	2.727	2.296	1.852	1.392	0.917	0.425
x6	0.000	0.000	0.000	0.000	0.000	0.000
x7	0.000	0.000	0.000	0.000	0.000	0.000
x8	4.675	4.675	4.675	4.675	4.675	4.675
x9	0.000	0.000	0.000	0.000	0.000	0.000
x10	0.067	0.145	0.224	0.307	0.392	0.481
x11	2.600	2.600	2.600	2.600	2.600	2.600
x12	3.900	3.900	3.900	3.900	3.900	3.900
x13	0.000	0.000	0.000	0.000	0.000	0.000
x14	18.174	17.835	17.485	17.123	16.749	16.361
x15	13.650	13.698	13.748	13.800	13.853	13.909
x16	6.765	6.813	6.863	6.915	6.968	7.024
MAG						
f1	0.604	0.606	0.609	0.611	0.613	0.616
f2	0.530	0.528	0.526	0.524	0.521	0.519
f3	5.243	5.215	5.185	5.155	5.122	5.091
F1	4.958	4.960	4.962	4.964	4.967	4.969
F2	1.338	1.336	1.334	1.332	1.330	1.328
F3	55.453	54.252	53.049	51.846	50.640	49.436
MAR						
f1	76.0%	76.3%	76.6%	76.9%	77.2%	77.6%
f2	76.0%	76.3%	76.6%	76.9%	77.2%	77.6%
f3	76.0%	76.3%	76.6%	76.9%	77.2%	77.6%
	f*	f-				
f1	0.781	0.045				
f2	0.372	1.029				
f3	2.898	12.668				

BANK ASSET AND LIABILITY MANAGEMENT
-- A FUZZY MATHEMATICAL PROGRAMMING MODEL

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AN ABSTRACT OF A MASTER'S THESIS

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ABSTRACT

Mathematical programming has been incorporated with bank asset and liability management in order to obtain an optimal structure of balance sheet position over a multiperiod planning horizon.

In managing bank balance-sheet position, profit, liquidity and their associated risk are the main concerns of financial strategy.

While dealing with the deterministic models, we suggest using an approach, Interactive Sequential Goal Programming II (ISGP II), to obtain a set of nondominated solutions with Maximum Achievable Rate (MAR) for three objectives. Through the trade-off procedure of ISGP II, we can find the solutions which are possible to reach under limited resources and acceptable by the decision makers.

Since the real-world situations are uncertain in nature, instead of expressing the imprecision with randomness as in most stochastic models, we employ the Fuzzy Set Theory to reformulate the deterministic bank model and use the concept of α -level set to handle the uncertainty of areas in which human judgement, evaluation and decision are important.

In our study, we conclude that when the α value of α -level set is given by decision makers, the optimal structure of our fuzzy bank model will be the one with ultra-optimistic coefficients in all objectives.

